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MEASURING LOAN RECOVERY RATE: METHODOLOGY AND EMPIRICAL EVIDENCE

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SUMMARY

This paper aims at proposing a new methodology to compute recovery rate on non-performing bank loans, in order to confine this variable within the interval [0,1]. Such a methodology is then applied to data on loans gathered by the Bank of Italy and some interesting characteristics of the loan recovery process in the Italian banking market are highlighted. The combined effects of some variables on the recovery rates are also analysed. In particular, the presence of either collateral or personal guarantee, the borrower's residence area are considered, thereby emphasizing the relationship between the recovery rate and the total exposure.

Keywords: *recovery rate, total exposure, boundary problem.*

1. INTRODUCTION

The Basel II Accord (Basel Committee on Banking Supervision BCBS, 2004) requires recovery rate estimates to compute the provision fair level needed to adjust the available capital amount. Recovery rate estimates are thus important inputs for credit risk models.

Different approaches have been proposed to compute bank loan recovery rates (see Altman et al., 2005a), but only few of them avoid both negative and greater than one values. By proposing the concept of ''total exposure'', this study suggests a new methodology to compute the bank loan recovery rate, in order to confine this variable within the interval [0,1]. The proposed measure is then applied and tested by using the data the Bank of Italy (Banca d'Italia, 2001) gathered on bank loans. Results are then compared with those achieved by Grippa et al. (2005), who applied a different measure to the same Bank of Italy's survey.

The present paper is organized as follows. The second section analyses the different approaches proposed to compute the recovery rate and the results obtained by their applications. Section 3 proposes the new methodology to compute the recovery rate. The fourth section presents the Bank of Italy's database, which the proposed

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Even if this work is the result of the close cooperation between the authors, sections 1 and 2 are mainly due to Michele Zenga while sections 3, 4 and 5 are due to Raffaella Calabrese.

methodology is applied to. The empirical evidence on recovery rates is then exhibited in section 5. Finally, section 6 is devoted to conclusions.

2. A LITERATURE REVIEW

The probability of default and the recovery rate are key variables in credit risk models. This paper focuses on the recovery rate (r), obtained as one minus Loss-Given-Default (LGD), which represents the lender's loss in the event of the borrower's default. The recovery rate plays an important role since the Basel II Accord requires LGD estimates under the advanced Internal-Rating-Based (IRB) approach[1] (BCBS, 2004, paragraph 286-317).

In the U.S. market, the priority rule in bankrupt occurrence is based on the capital structure of the firm, known as classification by ''seniority'' (Schuermann, 2004). The value of a defaulted firm has to be distributed first to senior creditors, following the sequence secured, unsecured and subordinated, and, only after they are fully satisfied, to subordinated and junior creditors.

Several studies considered recovery rates on corporate bonds (e.g. Altman, 1989, Renault and Scaillet, 2004 and Schuermann, 2004), while few authors dealt with bank loans (e.g. Asarnow and Edwards, 1995, Dermine and Neto de Carvalho, 2006 and Grippa et al. 2005). As is well known recovery rates on corporate bonds and on bank loans are significantly different. In particular, Schuermann (2004), Carty and Lieberman (1996) showed that the average recovery rate of bank loans is higher than the mean of bonds. The determinants of this difference were investigated by Dermine and Neto de Carvalho (2006).

Three main approaches have been so far proposed to compute the recovery rate. All of them consider the recovery rate as the ratio between the debt recovery and the Exposure-At-Default (EAD), i.e. the debt value when default occurs. These approaches differ in interpreting the numerator and the denominator of the recovery rate.

The first approach considers large corporate debts often traded on a secondary market, even after a failure. In particular, EAD is the face value of the debt and the recovery amount is the market value of the asset soon after the default event[2]. Hence, recovery rates are easily obtained and the choice of a discount rate for the recovery and for EAD is not required. The main disadvantage of this approach is that it cannot be applied to all different types of debts, since trading prices are not available for all defaulted bonds and loans. Moreover, the market value of defaulted debts changes

[1] The banks adopting the advanced IRB approach are allowed to use their own estimates of LGDs, provided that they can demonstrate to supervisors that their internal models are conceptually sound and consistent with their historical experience.

[2] The price of the debt has often been recorded roughly from 14 to 60 days after the default in the U.S. market, e.g. between two and eight weeks after the default in Carty and Lieberman's (1996) analysis.

according to the supply and the demand of assets. The trading price could thus be different from the actual recovery amount.

Renault and Scaillet (2004) applied such an approach to Standard & Poor's 623 bonds of different seniority defaulted from 1981 to 1999. On the contrary, Carty and Lieberman (1996) used it on assets of similar seniority. In particular, they analysed Moody's recovery rates on senior secured bank loans and bonds defaulted during the period 1991-1996 in the U.S. market.

Alternatively, in the second approach EAD is considered as the debt outstanding at the default event. During the reorganization process, other assets are offered to investors. The recovery amount is the value of offered assets or of security at the end of the reorganization process. The definition of the debt recovery is thus linked with a market price. This approach is known as ''ultimate recovery'' (Friedman and Sandow, 2003), since post-default debt recoveries are evaluated at the end of the reorganization process[3]. This framework cannot be applied to all different types of debts. EAD and the recovery concern different periods, so the choice of a specific discount rate is required. However, the increasing calculation complexity allows to supply more accurate estimate of the recovery rate.

Friedman and Sandow (2003) applied the ultimate recovery approach to about 1,400 Standard and Poor's data on U.S. loans and bonds defaulted since 1988. Moreover, Asarnow and Edwards (1995) analysed ultimate recoveries of 831 Citybank's U.S. corporate loans defaulted in the period 1970-1993.

In the third approach, EAD is considered as the outstanding debt at the time of default, like in the ultimate recovery approach. The recovery is the actual recovery amount from the default event to the end of the recovery process. Cash flows are opportunely discounted and the calculation is quite complex; however, only this approach can be applied to all different types of debts, i.e. to loans of low amounts that are usually not listed on the market.

Because of the complexity and the large number of information required, the last approach has been applied in few studies. For example Dermine and Neto de Carvalho (2006) analysed 374 Banco Commercial Portugês non-performing loans over the period 1995-2000.

Grippa et al. (2005) applied the last approach to the Bank of Italy's data (Banca d'Italia, 2001), which are analysed also in the present work. In particular, they computed the recovery rate as the ratio between the actual recovery amount (*RE*) and EAD

$$r = \frac{RE^C}{EAD^C}, \tag{2.1}$$

by capitalising both the variables at the same moment.

Otherwise, the Bank of Italy (Banca d'Italia, 2001) calculated the recovery rate

[3] Gupton and Stein (2002) obtained that the median of the reorganization time period is 1.75 years.

net of legal costs (L)

$$r = \frac{RE^C - L^C}{EAD^C}, \tag{2.2}$$

by subtracting capitalised legal costs L^C from the numerator of the recovery rate.

Finally, some considerations on these three approaches to measure the recovery rate are needed. Recovery rates based on trading prices shortly after default are overwhelmingly in the range [0,1], with rare occurrence slightly greater than one (Carty and Lieberman, 1996, Renault and Scaillet, 2004). On the contrary, the remaining approaches exhibit many more recovery rates greater than one, about 3.5% of observations for Friedman and Sandow (2003) and almost 20% for Carty and Lieberman (1996). Besides, the incidence of recovery rates equal to zero or one is much higher (Asarnow and Edwards, 1995, Dermine and Neto de Carvalho, 2006) in the last approaches.

However, a considerable influence on the recovery rate characteristics is shown by both the choices of the default definition and of the discount rate.

3. A METHODOLOGICAL PROPOSAL TO COMPUTE LOAN RECOVERY RATE

In order to compute LGD, the Basel II Accord established that loss has to be considered in an economic sense, and not in a mere accounting perspective[4]. The loss should thus include legal costs and interest on delayed payment, calculated from the default event to the recovery procedure closing. On the contrary, EAD does not include legal costs and interest on delayed payment, since the exposure is evaluated at the default statement. Indeed, the recovery amount could be higher than EAD to recoup legal costs and interest on delayed payment. Using the expression (2.1), the recovery rate could thus be greater than one. Therefore, LGD turns out to be negative, as the BCBS (2005) and the Bank of Italy (Banca d'Italia, 2006 pp.44) pointed out.

To represent the density function of the recovery rate, models and methodologies confine this variable within the interval [0,1], e.g. the beta density function (J.P. Morgan, 1997) and the beta kernel estimator (Renault and Scaillet, 2004). These frameworks cannot indeed be applied to Carty and Lieberman (1996), Friedman and Sandow (2003), Grippa et al. (2005) and Schuermann's (2004) analyses, which considered recovery rates greater than one. Moreover, some authors prefer to ignore recovery rates exceeding a given value, thus loosing some information, e.g. Friedman and Sandow (2003) and Grippa et al. (2005) did not consider values greater than 1.2.

The recovery rate net of legal costs, defined by the expression (2.2), could however assume either negative or greater than one values. In the latter case, from the expression (2.2), the capitalised recovery amount RE^C is higher than the sum of EAD^C

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4 ''Loss must include discount effects and material direct and indirect costs associated with collecting on the exposure'' (BCBS, 2004 paragraph 460).

and the capitalised legal costs L^C

$$RE^C > L^C + EAD^C. \tag{3.1}$$

Such an inequality shows that the recovery rate could be greater than one since the bank not only could cover the amount of the capitalised outstanding loan EAD^C plus the capitalised legal costs L^C, but also interest on delayed payment. In support of this remark, the Bank of Italy (Banca d'Italia, 2006 pp.44) explained that the condition (3.1) is due to the difference between the interest rate on delayed payment and the discount rate. Usually the former is higher than the latter.

In order to overcome the boundary problem, the present work suggests computing the loan recovery rate as the ratio between the capitalised actual recovery amount RE^C and the capitalised total exposure TE^C, where the latter variable is the sum of EAD, the capitalised legal costs L^C and the interest on delayed payment (I)

$$r = \frac{RE^C}{TE^C} = \frac{RE^C}{EAD + I + L^C}. \tag{3.2}$$

Following the Basel II Accord (BCBS, 2004 paragraph 460), but in contrast with the expressions (2.1) and (2.2), the proposed concept of total exposure TE^C includes the interest on delayed payment I and the capitalised legal costs L^C, since such amounts could be recovered and, thus, included in the numerator RE^C. EAD is not capitalised because the passing of time has just been considered by summing the interest on delayed payment.

Beside the boundary property, there are two main advantages of this methodology. The first is avoiding the problem due to the discrepancy between the discount rate and the interest rate on delayed payment. The second one is allowing the recovery rate to be evaluated also when EAD is null. This can happen just for debts thoroughly guaranteed by a collateral, as the Bank of Italy (Banca d'Italia, 2006 pp. 57) established.

4. THE SURVEY ON ITALIAN LOAN RECOVERY PROCESS

A comprehensive survey on the loan recovery process of Italian banks was conducted in the years 2000-2001 by the Bank of Italy. Its purpose was to gather information on the main characteristics of the Italian recovery process.

The survey is divided into three main sections. The first focuses on organizational aspects, the second regards credit recovery procedures and the last concerns analytical data on individual debts. About 250 banks were surveyed by means of a questionnaire. The database is composed of 169,778 defaulted borrowers and, since they covered nearly 90% of total domestic loans, this is the most recent and the most important survey on the Italian recovery process.

It is important to specify that data refer to individual loans which are privately held, and not listed on the market. In particular, loans are towards Italian resident debtors non-performing on the 31/12/1998 and entirely written off within the end of 1999.

The available information in the database includes many characteristics[5] of the loans. The analysis in the following section takes into account the presence of either personal guarantee or collateral, the borrower's residence area, in addition to the variables considered in the expression (3.2).

It is worth noting that the Bank of Italy applied the third above-mentioned approach. In fact, the exposure represents the outstanding debt at the time of default and the recovery is considered as the actual recovery amount.

The definition of default (Banca d'Italia, 2004 pp.II.10) chosen in the survey is narrower than the one proposed by the BCBS (2004, paragraph 452). The difference is the inclusion of transitory non-performing debts. Finally, the Bank of Italy established the discount rates for each quarter from 1975 to 1999 by relying on the interest rates on short-term loans.

5. EMPIRICAL RESULTS OF THE METHODOLOGICAL PROPOSAL

The methodology proposed in this work, given by the expression (3.2), is applied to the Bank of Italy's database. Only legal costs[6], and not their capitalised values are known, so the following approximation of the expression (3.2) can be applied

$$r = \frac{RE^C}{EAD + I + L}. \tag{5.1}$$

The Bank of Italy also processed data on the Italian recovery process (Banca d'Italia, 2001 and Grippa et al., 2005). In this section different results are shown since the expression (5.1) is applied in order to compute the recovery rate, whereas the Bank of Italy considered the ratio (2.1). Moreover, the database does not exactly coincide with the one examined by the Bank of Italy: debts exhibiting a recovery rate greater than one are not considered, so the sample size is $N = 149,378$. On the contrary, the Bank of Italy ignored recovery rates greater than 1.2. Inconsistent data are due to the lack of systematic archives on the recovery process when the survey was conducted.

In order to analyse the properties of the methodology proposed in this paper, the following results are compared with those achieved by the Bank of Italy.

At first, the arithmetic mean of the ratios $r_i = \frac{RE_i^C}{TE_i^C}$, with $i = 1, 2, ..., N$, is computed

$$\overline{r} = \frac{1}{N}\sum_{i=1}^{N} r_i = \frac{1}{N}\sum_{i=1}^{N}\frac{RE_i^C}{TE_i^C} = \frac{1}{N}\sum_{j=1}^{k} r_j\, n_j = 0.3842, \tag{5.2}$$

[5] In order to know all the variables gathered by the Bank of Italy, see Banca d'Italia (2001) and Grippa et al. (2005).

[6] Even if the Bank of Italy specified the discount rates, it is not possible to capitalise the legal costs since the periods these variable refer to are unknown.

where n_j is the frequency of the recovery rate r_j, with $j = 1, 2, ..., k$. The previous mean has just a probabilistic and not an economic meaning.

Even though Grippa et al. (2005) chose 1.2 as maximum recovery rate, in the Bank of Italy's processing the average recovery percentage 37% is lower than the mean value (5.2). This discordance could be due to the greater incidence of high recovery rates exhibited by the proposal (5.1). Several high recovery rates are not considered by using the expression (2.1) as they result greater than 1.2.

It is important to note that the previous values cannot be compared with the estimates established by the Basel II Accord (BCBS, 2004 paragraph 287-8) for these concern debts of similar seniority. Under the foundation IRB approach, LGD is fixed at 45% for all senior unsecured debts and at 75% for all subordinated exposures. The BCBS's estimates are consistent with the average percentages of recovery 65.21% and 79% computed on senior loans by Asarnow and Edwards (1995) and on secured senior loans by Carty and Lieberman (1996).

The mode of the recovery rate distribution is the extreme value zero, with 23% of the observations. Besides, the recovery rate equal to one exhibits also a high percentage (7.7%), which becomes just 0.372% after the expression (2.1). Hence, the proposal of this work allows to achieve a higher incidence of the recovery rates equal to one compared to the expression (2.1).

Many authors obtained a bimodal recovery rate distribution. For instance, in Friedman and Sandow's (2003) study about 10% and 20% of the observations showed recovery rates respectively equal to zero and one. In a similar way, Asarnow and Edwards (1995) identified a higher frequency of recovery rates equal to one than the one of null recovery rates, since the authors considered only senior loans.

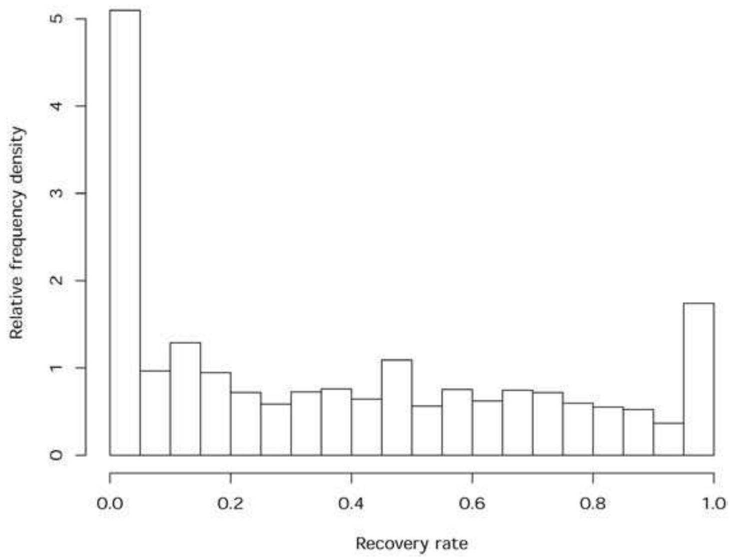

FIGURE 1. - *Relative frequency distribution of the recovery rates*

As the median recovery rate is 0.3333, the index of the asymmetry direction results

$$2[\bar{r} - Me(r)] = 0.1018,$$

thus the recovery rate distribution is positively asymmetric, as displayed in figure 1. On the contrary, Asarnow and Edwards (1995), Carty and Lieberman (1996) obtained a recovery rate distribution with negative asymmetry.

Afterwards, to investigate the recovery rate variability, the average absolute deviation $S_{M_1}(r)$ and the standard deviation $\sigma(r)$ are calculated

$$S_{M_1}(r) = \frac{\sum_{j=1}^{k} |r_j - \bar{r}| n_j}{N} = 0.2986$$

$$\sigma(r) = \sqrt{\frac{\sum_{j=1}^{k} (r_j - \bar{r})^2 n_j}{N}} = 0.3400.$$

The comparisons of the former values with the average recovery rate $\bar{r}$ allow to obtain relative variability indices

$$\frac{S_{M_1}(r)}{\bar{r}} = 066.7720 \qquad \frac{\sigma(r)}{\bar{r}} = 0.8850. \tag{5.3}$$

Both these values show that the dispersion of the recovery rate is significantly high. In particular, from the coefficient of variation the standard deviation accounts for 88.50% of the average recovery rate. Such a high variability could be due to the considerable incidence of the recovery rates equal to zero and one.

The coefficient of variation 0.9733, computed by Grippa et al. (2005), is higher than the correspondent value reported in the expression (5.3), since the Bank of Italy also considered recovery rates exceeding one.

The coefficients of variation computed by Carty and Lieberman (1996) (36.71% of the average recovery rate) and by Renault and Scaillet (2004) (60.31% of the average recovery rate) exhibit lower dispersions of the recovery rate than the Italian variability. In fact, in the previous analyses the incidence of recovery rates equal to zero or one is lower than the Italian.

5.1 *A disaggregate analysis*

In order to analyse the combined effect of different factors on recovery rates, an analysis is performed on the sub-sample of N_1=147,866 observations proving the complete range of information about the presence of either collateral or personal guarantee, the borrower's residence area and the capitalised total exposure[7] TE^C.

[7] The extreme values of TE^C classes are chosen in order to roughly achieve the same number of observations in each group.

As reported in the last row of table A.1, at the end of this paper, Islands show the lowest percentage (11.27%) of defaulted debts. This result disagrees with the expectations, but it is consistent with the highest percentage of the secured loans for Islands (10.22%), as table A.3 shows. It could be inferred that banks require either collateral or personal guarantee from the borrowers residing in Islands, where the economic situation is quite difficult. This practice allows the recovery of considerable amounts. For this reason the average recovery rate in Islands (0.4146) is exceeded only by the one (0.4165) of the south of Italy, as table A.4 exhibits.

Even if in the south of Italy and in Islands the economic conditions are similar, banks are less restrictive on the request of collateral or personal guarantee from the borrowers residing in South Italy. In table A.3, the rate of secured debts for South (4.19%) is different from the one for Islands, but it is similar to those showed by the rest of Italy. In spite of this, banks are able to recover considerable sums from the borrowers residing in South, thus obtaining the highest recovery rate mean for such area, as above-mentioned. Grippa et al. (2005) explain that in the south of Italy private agreements are widely used, thus reducing the duration of the recovery procedure.

The loan selection is not restrictive for the borrowers residing in the central area, which exhibits the highest percentage (31.42%) of defaulted debts and, consistently, the lowest one (1.60%) of secured loans, as tables A.1 and A.3 show. In spite of these characteristics, the average recovery rate for Central Italy is quite considerable (0.4095). This mean is higher than those for North-East (0.3493) and for North-West (0.3333), as table A.4 shows.

It is interesting to note from table A.2 that, in the whole Italian territory, the rate of the secured debts exhibits a rise as the total exposure increases. Such rise becomes considerable for the last TE^C class, passing from 11.53% to 22.24%. Also the partial distributions of the secured loans in each area show analogous increasing trends. An exception is represented by South Italy, where such trend becomes decreasing for the last two TE^C classes, as table A.2 exhibits.

Besides, the presence of either collateral or personal guarantee strongly affects the average recovery rate, which is 0.5666 for secured loans and 0.3798 for unsecured debts in the whole Italian territory, as table A.4 reports. The same table shows that in every Italian area the partial mean recovery rates of the secured debts present a constant trend with some fluctuations, which are stressed in the central area, as the total exposure increases. On the contrary, the conditional mean trend of the unsecured debts is evidently increasing with the rise of the capitalised total exposure in every area.

Finally, the analysis deals with the conditional variability of the recovery rate. In tables A.5 and A.6 the most interesting result is the decreasing partial dispersion of the recovery rate with the rise of the total exposure. In particular, this characteristic is satisfied by both secured and unsecured conditional recovery rates, computed in every area.

From the marginal recovery rate dispersions, which are reported in the last rows of tables A.5 and A.6, the north-west of Italy is the area with the highest dispersion around the average recovery rate. The coefficient of variation is 1.0327 and the ratio between the arithmetic mean of the absolute deviations and the average recovery rate is 0.8962. Also North-East shows a high dispersion of the recovery rate: the previous indices are respectively 0.9906 and 0.8697. The lowest conditional variability results in Central Italy, with a coefficient of variation and $\frac{S_{M_1}(r)}{\bar{r}}$ equal respectively to 0.7768 and 0.6747.

Finally, tables A.5 and A.6 show the recovery rates of the unsecured debts present a significant higher variability - with a coefficient of variation equal to 0.8918 - than the one (0.5372) exhibited by the recovery rates of the secured debts in the whole Italian territory. The previous result agrees with those obtained by Grippa et al. (2005), Renault and Scaillet (2004).

5.2 *The relationship between the recovery rate and the (total) exposure*

Several studies (e.g. Asarnow and Edwards, 1995, Carty and Liebermann, 1996, Dermine and Neto de Carvalho, 2006 and Grippa et al., 2005) showed disagreeing results concerning the exposure effect on the recovery rate. The analysis of this topic relies on the Bank of Italy's database, in particular on the sub-sample of $N_1 = 147,866$ observations considered in the former subsection.

At first, the exposure-weighted average recovery rate is computed as the ratio between the mean capitalised recovery amount and the mean capitalised total exposure

$$\bar{r}_{TE^C} = \frac{\overline{RE^C}}{\overline{TE^C}} = \frac{\sum_{i=1}^{N_1} RE_i^C}{\sum_{i=1}^{N_1} TE_i^C} = \frac{\sum_{i=1}^{N_1} r_i \, TE_i^C}{\sum_{i=1}^{N_1} TE_i^C}$$

$$= \frac{\sum_{j=1}^{k} \left[r_j \sum_{l=1}^{n_j} TE_l^C \right]}{\sum_{i=1}^{N_1} TE_i^C} = 0.3880.$$

The Basel II Accord (BCBS, 2004 paragraph 634) suggests the previous mean calculation because of its economic meaning.

From the last row of table 1, the average recovery rate computed on the above-mentioned sub-sample is 0.3846, so it is lower than $\bar{r}_{TE^C}$. The linear correlation between the recovery rate and TE^C is thus positive, as Calabrese (2007) has proved.

Moreover, by the linear correlation coefficient between the recovery rate and TE^C

$$\rho(r, TE^C) = \frac{\frac{1}{N_1} \sum_{j=1}^{k} \left[r_j \sum_{l=1}^{n_j} TE_l^C \right] - \bar{r}\overline{TE^C}}{\sigma(r) \;\; \sigma(TE^C)}$$

$$= \frac{555.4166}{0.3395 \cdot 10,447,512} = 0.00016 \qquad (5.4)$$

the intensity of the linear correlation between such variables should be negligible.

TABLE 1. - *Recovery rates by capitalised total exposure TE^C*

TE^C	*Frequency*	$\bar{r}_h$	$\frac{\sigma_h(r)}{\bar{r}_h}$	$\frac{S^h_{M_1}(r)}{\bar{r}_h}$
(0;547]	6,614	0.2961	1.5302	1.3921
(547;1,549]	6,104	0.3071	1.3380	1.2051
(1,549;2,066]	5,447	0.3105	1.2544	1.0889
(2,066;2,582]	5,029	0.3236	1.1653	1.0189
(2,582;3,615]	8,655	0.3510	1.0302	0.9034
(3,615;4,648]	7,428	0.3635	0.9582	0.8421
(4,648;5,681]	6,823	0.3747	0.9277	0.8209
(5,681;6,714]	5,995	0.3899	0.8705	0.7664
(6,714;8,263]	7,935	0.3931	0.8499	0.7466
(8,263;9,813]	6,712	0.3978	0.8223	0.7242
(9,813;11,362]	5,720	0.3974	0.8221	0.7272
(11,362;13,428]	6,722	0.3900	0.8118	0.7115
(13,428;16,010]	7,210	0.3913	0.7984	0.6977
(16,010;19,109]	6,865	0.3853	0.8048	0.7057
(19,109;23,241]	6,908	0.3942	0.7859	0.6862
(23,241;28,922]	6,812	0.3987	0.7901	0.6907
(28,922;38,218]	7,219	0.4129	0.7653	0.6687
(38,218;52,162]	6,694	0.4305	0.7236	0.6260
(52,162;74,886]	6,705	0.4429	0.6988	0.6010
(74,886;119,302]	6,736	0.4338	0.6867	0.5855
(119,302;247,282]	6,777	0.4336	0.6727	0.5715
>247,282	6,756	0.4148	0.6873	0.5788
Total	147,866	0.3846	0.8827	0.7748

On the contrary, the difference between the simple and the exposure-weighted means becomes significant in the Bank of Italy's processing[8]: by using the expression (2.1) EAD^C is considered as a contributory factor to the recovery rates. Besides, as the simple mean is higher than the exposure-weighted average recovery rate, large loan size exhibits lower recovery rates. Thus, the expressions (2.1) and (5.1) allow to get different relationships of the linear dependence between the exposure and the recovery rate.

Some studies lead to different results on this topic: Asarnow and Edwards (1995), Carty and Lieberman (1996) found no significant relation between LGD and the loan size, instead Dermine and Neto de Carvalho (2006) hit upon a negative correlation between the same variables.

[8] In particular, the simple and the exposure-weighted average recovery rates are respectively 37.5% and 27.6% (Banca d'Italia, 2001).

In order to investigate the relationship between TE^C and the recovery rate, the dependence of the conditional mean recovery rates given a TE^C interval is analysed. From table 1 the conditional average recovery rates given a TE^C interval show an increasing trend, except for some fluctuations for the TE^C classes from 9,813 to 19,106 euros and from 74,886 euros to the last class.

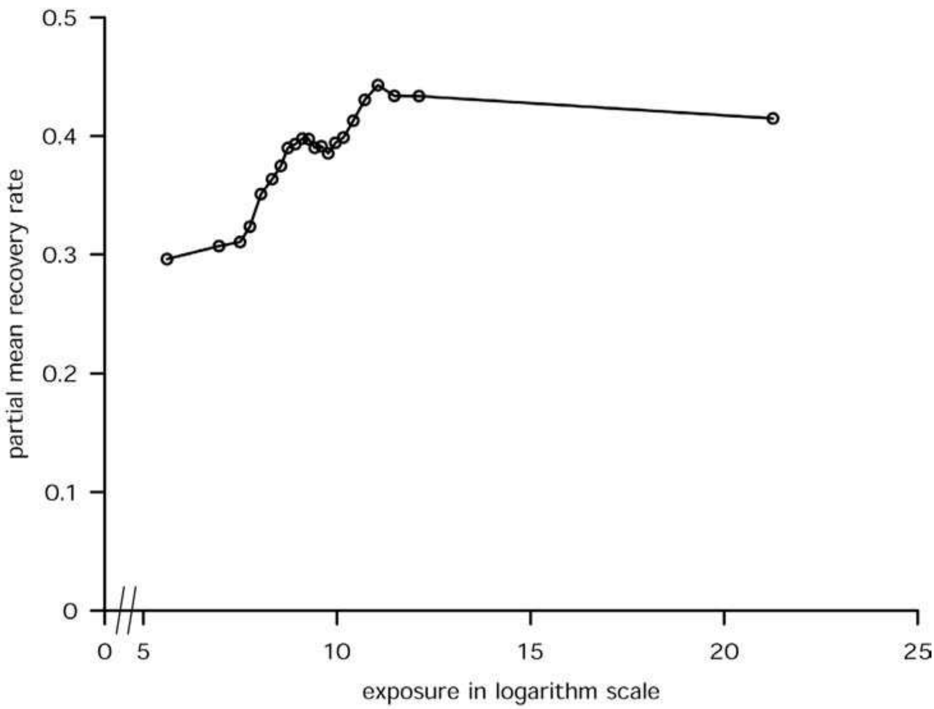

FIGURE 2. - *The broken-line regression of the recovery rates*

To measure the dependence of the conditional mean recovery rates given a TE^C interval, Pearson's correlation ratio is computed

$$\eta_{(r|TE^C)} = \sqrt{\frac{\sum_{h=1}^{22} [\bar{r}_h - \bar{r}]^2 n_{\cdot h}}{N_1 \ \sigma^2(r)}} = \sqrt{0.02062} = 0.14360,$$

where $\bar{r}_h$ is the partial mean of the TE^C h-th class and $n_{\cdot h}$ is the frequency of the TE^C h-th class, with $h = 1, 2, ..., 22$. It is inferred that 2.062% of the total variance is due to the variance of the partial means $\bar{r}_h$, with $h = 1, 2, ..., 22$. Thus, the conditional mean recovery rates given a TE^C class should show a very low dependence.

The broken-line regression, passing through the partial means $\bar{r}_h$, with $h = 1, 2, ..., 22$, is graphed by figure 2. The dependence of the conditional mean recovery rates represented is not very low. Hence, since the variability within the groups is so high that Pearson's correlation ratio results low even though the dependence of the partial means $\bar{r}_h$, with $h = 1, 2, ..., 22$, is not.

From this consideration it could be inferred also that the intensity of the linear correlation between the recovery rate and TE^C is not negligible. The linear correlation coefficient between such variables is close to zero because of the high variabilities of r and TE^C, as the expression (5.4) exhibits.

By computing the following index

$$I_d^{*2} = \frac{\rho^2_{(r,TE^C)}}{\eta^2_{(r|TE^C)}} = \frac{0.00016^2}{0.14360^2} = 0.000001,$$

the weighted regression of the partial means explains only a negligible part of the conditional mean-variance.

Moreover, figure 2 shows that partial mean recovery rates are increasing with the TE^C rise. This result disagrees with the one by Grippa et al. (2005): the partial average recovery rate is 0.39 for EAD^Cs lower than 75,000 euros and the conditional mean is 0.23 for the exposures higher than 500,000 euros. Hence, it could be asserted that the expressions (2.1) and (5.1) allow to get different dependence relationships between the average recovery rate and the loan size.

In contrast with the previous results, Carty and Lieberman (1996), Asarnow and Edwards (1995) obtained negligible differences among the condition average recovery rates given a loan size class[9].

Finally, the last two columns of table A.1 allow to note that the trend of the conditional recovery rate variability is decreasing with the TE^C rise. The exceptions are some fluctuations for the TE^C classes from 16,010 to 23,241 euros and for the last class, if considering the coefficient of variation, and for the TE^C classes from 9,813 to 11,362 euros, from 16,010 to 23, 241 euros and for the last class if analysing $\frac{S^h_{M_1}(r)}{\bar{r}_h}$, with $h = 1, 2, ..., 22$. On the contrary, in Carty and Lieberman's (1996) analysis recovery rate variabilities within each debt size group are similar.

6. CONCLUDING REMARKS

In the present work a new methodology to compute the recovery rate is proposed that overcomes the boundary problem by means of the ''total exposure'' concept. Such a methodology has been applied to a sample of roughly 150,000 non-perfor-

9 In particular, in Carty and Lieberman's (1996) analysis the partial average recovery rates is respectively 0.79 on all firms, 0.78 on firms with assets exceeding 25 million dollars, 0.77 on firms concerning exposures higher than 50 million dollars and 0.77 on debts with assets above 100 million dollars.

ming loans in the Italian banking market at the end of 1998 and written off within the end of 1999. The average percentage of recovery is 38.42%. Afterwards, the determinants of the recovery rate are analysed, leading to the following results.

The partial average recovery rates concerning the south of Italy or Islands are the highest. For the latter area this characteristic is explained by the bank request of either collateral or personal guarantee. On the contrary, even though the incidence of the secured loans is the lowest in Central Italy, the partial average recovery rate for such a zone is quite considerable. Islands and the central area exhibit respectively the lowest and the highest percentage of all the defaulted debts.

The presence of either collateral or personal guarantee strongly affects the mean recovery rate. Moreover, the mean recovery rate trend of the unsecured debts is increasing with the rise of the total exposure. On all the defaulted loans, the conditional recovery rate means and variabilities, given a TE^C class, show respectively increasing and decreasing trends with the total exposure rise.

Three main conclusions could be drawn from this empirical study. First, the recovery rate shows a bimodal distribution, with a high incidence of the recovery rates equal to either zero (23%) or one (7.7%). Loan portfolio models should capture this characteristic. Second, the total exposure, the presence of either collateral or personal guarantee and the borrower's residence area are contributory factors to the recovery rates. Third, the relationship between the recovery rate and the exposure changes according to whether legal costs are included or not in the exposure amount.

The results of the present work could be useful in the joint analysis of the recovery and the default risks, about which disagreeing results are showed (Altman et al., 2005b and J. P. Morgan, 1997). Moreover, it is interesting to consider the interactions between the recovery and the default rates in a dynamic framework. Using such approach, Bruche and González-Aguado (2008) obtain a much higher credit risk than the one achieved in a static model.

RIASSUNTO

L'obiettivo di questo lavoro consiste nel proporre una nuova metodologia per calcolare il tasso di recupero dei prestiti bancari dichiarati inadempienti, affinchè tale variabile sia definita nell'intervallo [0,1]. Tale proposta è applicata, quindi, ai dati raccolti dalla Banca d'Italia sui prestiti bancari, ottenendo così interessanti risultati sulle caratteristiche dell'attività di recupero crediti del sistema bancario italiano. Si analizzano, in particolare, gli effetti congiunti sui tassi di recupero prodotti dalla presenza di garanzie reali o personali e dall'area geografica di residenza del debitore, attribuendo notevole rilevanza al legame che intercorre tra il tasso di recupero e l'esposizione totale.

REFERENCES

Altman E., Resti A., Sironi A. (2005a). Loss Given Default: A Review of the Literature. *The Next Challenge in Credit Risk Management*. Altman E.I., Resti A., Sironi A. (eds.), Riskbooks, London.

Altman E.I., Brady B., Resti A., Sironi A. (2005b). The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications. *Journal of Business*, **78**, 6, 2203-2227.

Altman E. (1989). Measuring Corporate Bond Mortality and Performance. *Journal of Finance*, **28**, 909-922.

Asarnow E., Edwards D. (1995). Measuring Loss on Default Bank Loans: A 24-Year Study. *Journal of Commercial Lending*, **77**, 7, 11-23.

Banca d'Italia (2001). Principali risultati della rilevazione sull'attività di recupero dei crediti. *Bollettino di Vigilanza*, **12**, December.

Banca d'Italia (2004). Centrale dei Rischi. Istruzioni per gli Intermediari Creditizi. *Normativa di Vigilanza sulle Banche*. June.

Banca d'Italia (2006). Recepimento della nuova regolamentazione prudenziale internazionale. Metodo dei rating interni per il calcolo del requisito patrimoniale a fronte del rischio di credito. Documenti Basilea 2. *Funzioni di Vigilanza*. June.

Basel Committee on Banking Supervision (2005). *Guidance on Paragraph 468 of the Framework Document*. July, Basilea, BIS.

Basel Committee on Banking Supervision (2004). *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. June, Basilea, BIS.

Bruche M., González-Aguado C. (2008). Recovery Rates, Default Probabilities and the Credit Cycle. *CEMFI*, working paper.

Calabrese R. (2007). *Default and Recovery Risk*. PhD Thesis. Supervisor: Professor Michele Zenga. Bicocca University of Milan.

Carty L., Lieberman D. (1996). *Defaulted Bank Loan Recoveries*. Moody's Special Comment, November.

Dermine J., Neto de Carvalho C. (2006). Bank loan losses-given-default: a case study. *Journal of Banking and Finance*, **30**, 4, 1219-1243.

Friedman C., Sandow S. (2003). Ultimate recoveries. *Risk*, **16**, 8, 69-73.

Grippa P., Iannotti S., Leandri F. (2005). Recovery Rates in the Banking: Stylised Facts Emerging from Italian Experience. Recovery Risk. *The Next Challenge in Credit Risk Management*. Altman E.I., Resti A., Sironi A. (eds.), Riskbooks, London.

Gupton G.M., Stein R.M. (2002). *Losscalc^{TM}: model for predicting loss given default (LGD)*. Moody's Investors Service.

J.P. Morgan (1997). *CreditMetrics*. Technical Document, New York.

Renault O., Scaillet O. (2004). On the way to recovery: A nonparametric bias free estimation of recovery rate densities. *Journal of Banking and Finance*, **28**, 12, 1219-1243.

Schuermann T. (2004). What Do We Know About Loss Given Default? Recovery Risk. *Wharton Financial Institutions Center Working Paper*. n. 04-01.

TABLE A.1 - *Partial frequencies of defaulted loans by total exposure, presence of either collateral or personal guarantee and borrower's residence area*

	North-West		North-East		Central Italy		South		Islands		Total		Total
TE^C	secured	unsecured	secured	unsecured	secured	unsecured	secured	unsecured	secured	unsecured	secured	unsecured	
(0;547]	7	1,592	8	1,242	5	1,494	9	1,508	9	740	38	6,576	6,614
(547;1,549]	5	1,488	4	1,148	3	1,543	4	1,272	12	625	28	6,076	6,104
(1,549;2,066]	7	1,390	9	986	1	1,353	11	1,079	14	597	42	5,405	5,447
(2,066;2,582]	3	1,164	6	989	8	1,287	26	1,013	8	525	51	4,978	5,029
(2,582;3,615]	28	2,068	11	1,577	7	2,218	23	1,719	16	988	85	8,570	8,655
(3,615;4,648]	13	1,654	15	1,222	6	2,056	38	1,479	11	934	83	7,345	7,428
(4,648;5,681]	17	1,511	7	1,119	11	1,932	34	1,304	12	876	81	6,742	6,823
(5,681;6,714]	15	1,372	15	895	9	1,744	32	1,132	11	770	82	5,913	5,995
(6,714;8,263]	18	1,658	18	1,196	11	2,305	47	1,559	37	1,086	131	7,804	7,935
(8,263;9,813]	25	1,370	13	1,015	16	1,969	23	1,320	23	938	100	6,612	6,712
(9,813;11,362]	27	1,184	14	862	20	1,671	33	1,149	22	738	116	5,604	5,720
(11,362;13,428]	28	1,383	28	981	19	2,070	37	1,310	45	821	157	6,565	6,722
(13,428;16,010]	34	1,421	35	983	22	2,334	52	1,365	50	914	193	7,017	7,210
(16,010;19,109]	48	1,373	48	930	26	2,248	43	1,270	57	822	222	6,643	6,865
(19,109;23,241]	49	1,391	47	905	29	2,276	72	1,244	70	825	267	6,641	6,908
(23,241;28,922]	56	1,341	69	998	35	2,272	71	1,187	79	704	310	6,502	6,812
(28,922;38,218]	107	1,548	75	1,122	49	2,504	97	1,137	83	497	411	6,808	7,219
(38,218;52,162]	96	1,423	73	1,061	71	2,402	102	978	82	406	424	6,270	6,694
(52,162;74,886]	137	1,452	105	1,051	84	2,423	114	952	95	292	535	6,170	6,705
(74,886;119,302]	165	1,436	108	1,029	90	2,481	119	947	92	269	574	6,162	6,736
(119,302;247,282]	195	1,361	117	960	101	2,599	90	881	181	292	684	6,093	6,777
>247,282	246	1,173	174	656	122	2,534	85	774	693	299	1,320	5,436	6,756
Total	1,326	31,753	999	22,927	745	45,715	1,162	26,579	1,702	14,958	5,934	141,932	147,866
	33,079		23,926		46,460		27,741		16,660		147,866		
	0.2237		0.1618		0.3142		0.1876		0.1127		1		

TABLE A.2 - *Conditional relative frequencies of defaulted loans given the borrower's residence area ad given the presence/absence of either collateral or personal guarantee by total exposure*

	North-West		North-East		Central Italy		South		Islands		Italy	
TE^C	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**
(0;547]	0.0053	0.0501	0.0080	0.0542	0.0067	0.0327	0.0077	0.0567	0.0053	0.0495	0.0064	0.0463
(547;1,549]	0.0038	0.0469	0.0040	0.0501	0.0040	0.0338	0.0034	0.0479	0.0071	0.0418	0.0047	0.0428
(1,549;2,066]	0.0053	0.0438	0.0090	0.0430	0.0013	0.0296	0.0095	0.0406	0.0082	0.0399	0.0071	0.0381
(2,066;2,582]	0.0023	0.0367	0.0060	0.0431	0.0107	0.0282	0.0224	0.0381	0.0047	0.0351	0.0086	0.0351
(2,582;3,615]	0.0211	0.0651	0.0110	0.0688	0.0094	0.0485	0.0198	0.0647	0.0094	0.0661	0.0143	0.0604
(3,615;4,648]	0.0098	0.0521	0.0150	0.0533	0.0081	0.0450	0.0327	0.0556	0.0065	0.0624	0.0140	0.0518
(4,648;5,681]	0.0128	0.0476	0.0070	0.0488	0.0148	0.0423	0.0293	0.0491	0.0071	0.0586	0.0137	0.0475
(5,681;6,714]	0.0113	0.0432	0.0150	0.0390	0.0121	0.0381	0.0275	0.0426	0.0065	0.0515	0.0138	0.0417
(6,714;8,263]	0.0136	0.0522	0.0180	0.0522	0.0148	0.0504	0.0404	0.0587	0.0217	0.0726	0.0221	0.0550
(8,263;9,813]	0.0189	0.0431	0.0130	0.0443	0.0215	0.0431	0.0198	0.0497	0.0135	0.0627	0.0169	0.0466
(9,813;11,362]	0.0204	0.0373	0.0140	0.0376	0.0268	0.0366	0.0284	0.0432	0.0129	0.0493	0.0195	0.0395
(11,362;13,428]	0.0211	0.0436	0.0280	0.0428	0.0255	0.0453	0.0318	0.0493	0.0264	0.0549	0.0265	0.0463
(13,428;16,010]	0.0256	0.0448	0.0350	0.0429	0.0295	0.0511	0.0448	0.0514	0.0294	0.0611	0.0325	0.0494
(16,010;19,109]	0.0362	0.0432	0.0480	0.0406	0.0349	0.0492	0.0370	0.0478	0.0335	0.0550	0.0374	0.0468
(19,109;23,241]	0.0370	0.0438	0.0470	0.0395	0.0389	0.0498	0.0620	0.0468	0.0411	0.0552	0.0450	0.0468
(23,241;28,922]	0.0422	0.0422	0.0691	0.0435	0.0470	0.0497	0.0611	0.0447	0.0464	0.0471	0.0522	0.0458
(28,922;38,218]	0.0807	0.0488	0.0751	0.0489	0.0658	0.0548	0.0835	0.0428	0.0488	0.0332	0.0693	0.0480
(38,218;52,162]	0.0724	0.0448	0.0731	0.0463	0.0953	0.0525	0.0878	0.0368	0.0482	0.0271	0.0715	0.0442
(52,162;74,886]	0.1033	0.0457	0.1051	0.0458	0.1128	0.0530	0.0981	0.0358	0.0558	0.0195	0.0902	0.0435
(74,886;119,302]	0.1244	0.0452	0.1081	0.0449	0.1208	0.0543	0.1024	0.0356	0.0541	0.0180	0.0967	0.0434
(119,302;247,282]	0.1471	0.0429	0.1171	0.0419	0.1356	0.0569	0.0775	0.0331	0.1063	0.0195	0.1153	0.0429
>247,282	0.1855	0.0369	0.1742	0.0286	0.1638	0.0554	0.0731	0.0291	0.4072	0.0200	0.2224	0.0383
Total	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

TABLE A.3 - *Conditional relative frequencies of defaulted loans given the borrower's residence area by total exposure and presence of either collateral or personal guarantee*

	North-West		North-East		Central Italy		South		Islands		Italy	
TE^C	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**
(0;547]	0.0002	0.0481	0.0003	0.0519	0.0001	0.0322	0.0003	0.0544	0.0005	0.0444	0.0003	0.0445
(547;1,549]	0.0002	0.0450	0.0002	0.0480	0.0001	0.0332	0.0001	0.0459	0.0007	0.0375	0.0002	0.0411
(1,549;2,066]	0.0002	0.0420	0.0004	0.0412	0.0000	0.0291	0.0004	0.0389	0.0008	0.0358	0.0003	0.0366
(2,066;2,582]	0.0001	0.0352	0.0003	0.0413	0.0002	0.0277	0.0009	0.0365	0.0005	0.0315	0.0003	0.0337
(2,582;3,615]	0.0008	0.0625	0.0005	0.0659	0.0002	0.0477	0.0008	0.0620	0.0010	0.0593	0.0006	0.0580
(3,615;4,648]	0.0004	0.0500	0.0006	0.0511	0.0001	0.0443	0.0014	0.0533	0.0007	0.0561	0.0006	0.0497
(4,648;5,681]	0.0005	0.0457	0.0003	0.0468	0.0002	0.0416	0.0012	0.0470	0.0007	0.0526	0.0005	0.0456
(5,681;6,714]	0.0005	0.0415	0.0006	0.0374	0.0002	0.0375	0.0012	0.0408	0.0007	0.0462	0.0006	0.0400
(6,714;8,263]	0.0005	0.0501	0.0008	0.0500	0.0002	0.0496	0.0017	0.0562	0.0022	0.0652	0.0009	0.0528
(8,263;9,813]	0.0008	0.0414	0.0005	0.0424	0.0003	0.0424	0.0008	0.0476	0.0014	0.0563	0.0007	0.0447
(9,813;11,362]	0.0008	0.0358	0.0006	0.0360	0.0004	0.0360	0.0012	0.0414	0.0013	0.0443	0.0008	0.0379
(11,362;13,428]	0.0008	0.0418	0.0012	0.0410	0.0004	0.0446	0.0013	0.0472	0.0027	0.0493	0.0011	0.0444
(13,428;16,010]	0.0010	0.0430	0.0015	0.0411	0.0005	0.0502	0.0019	0.0492	0.0030	0.0549	0.0013	0.0475
(16,010;19,109]	0.0015	0.0415	0.0020	0.0389	0.0006	0.0484	0.0016	0.0458	0.0034	0.0493	0.0015	0.0449
(19,109;23,241]	0.0015	0.0421	0.0020	0.0378	0.0006	0.0490	0.0026	0.0448	0.0042	0.0495	0.0018	0.0449
(23,241;28,922]	0.0017	0.0405	0.0029	0.0417	0.0008	0.0489	0.0026	0.0428	0.0047	0.0423	0.0021	0.0440
(28,922;38,218]	0.0032	0.0468	0.0031	0.0469	0.0011	0.0539	0.0035	0.0410	0.0050	0.0298	0.0028	0.0460
(38,218;52,162]	0.0029	0.0430	0.0031	0.0443	0.0015	0.0517	0.0037	0.0353	0.0049	0.0244	0.0029	0.0424
(52,162;74,886]	0.0041	0.0439	0.0044	0.0439	0.0018	0.0522	0.0041	0.0343	0.0057	0.0175	0.0036	0.0417
(74,886;119,302]	0.0050	0.0434	0.0045	0.0430	0.0019	0.0534	0.0043	0.0341	0.0055	0.0161	0.0039	0.0417
(119,302;247,282]	0.0059	0.0411	0.0049	0.0401	0.0022	0.0559	0.0032	0.0318	0.0109	0.0175	0.0046	0.0412
>247,282	0.0074	0.0355	0.0073	0.0274	0.0026	0.0545	0.0031	0.0279	0.0416	0.0179	0.0089	0.0368
Total	**0.0401**	**0.9599**	**0.0418**	**0.9582**	**0.0160**	**0.9840**	**0.0419**	**0.9581**	**0.1022**	**0.8978**	**0.0401**	**0.9599**
	1		**1**		**1**		**1**		**1**		**1**	

TABLE A.4 - *Partial avarage recovery rates by total exposure, presence of either collateral or personal guarantee and borrower's residence area*

	North-West		**North-East**		**Central Italy**		**South**		**Islands**		**Italy**		**Total**
TE^C	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	
(0;547]	0.5714	0.2481	0.5000	0.2429	0.2000	0.3284	0.6667	0.3216	0.4444	0.3610	0.5000	0.2949	**0.2961**
(547;1,549]	0.6000	0.2848	0.5000	0.2505	0.8333	0.3194	0.5000	0.3400	0.5833	0.3536	0.5893	0.3058	**0.3071**
(1,549;2,066]	0.6190	0.2726	0.5556	0.2559	1.0000	0.3218	0.6970	0.3695	0.7619	0.3309	0.6825	0.3076	**0.3105**
(2,066;2,582]	0.5833	0.2758	0.7917	0.2710	0.3125	0.3368	0.6154	0.3653	0.7188	0.3892	0.6029	0.3208	**0.3236**
(2,582;3,615]	0.5893	0.2900	0.6576	0.3011	0.4381	0.3652	0.5420	0.3927	0.8167	0.4312	0.6157	0.3484	**0.3510**
(3,615;4,648]	0.6181	0.3014	0.4952	0.3063	0.6518	0.3607	0.7754	0.4244	0.6964	0.4294	0.6807	0.3599	**0.3635**
(4,648;5,681]	0.6203	0.3214	0.8921	0.3373	0.3707	0.3763	0.7369	0.4059	0.6009	0.4382	0.6560	0.3713	**0.3747**
(5,681;6,714]	0.6864	0.3417	0.4732	0.3513	0.5926	0.3929	0.6463	0.4313	0.6680	0.4283	0.6190	0.3867	**0.3899**
(6,714;8,263]	0.5774	0.3485	0.5981	0.3385	0.4952	0.4107	0.6966	0.4190	0.5764	0.4202	0.6158	0.3894	**0.3931**
(8,263;9,813]	0.6160	0.3572	0.5355	0.3808	0.6027	0.4098	0.6336	0.4148	0.6186	0.4041	0.6080	0.3947	**0.3978**
(9,813;11,362]	0.5604	0.3547	0.6582	0.3650	0.4443	0.4128	0.6082	0.4169	0.7314	0.4066	0.5982	0.3932	**0.3974**
(11,362;13,428]	0.4015	0.3478	0.5949	0.3420	0.6510	0.4178	0.6383	0.4050	0.6099	0.3880	0.5817	0.3854	**0.3900**
(13,428;16,010]	0.5492	0.3355	0.5965	0.3650	0.5154	0.4110	0.6988	0.4071	0.5992	0.3869	0.6072	0.3854	**0.3913**
(16,010;19,109]	0.5151	0.3406	0.5618	0.3559	0.5718	0.4183	0.7149	0.3770	0.5940	0.3602	0.5908	0.3784	**0.3853**
(19,109;23,241]	0.5298	0.3298	0.6546	0.3634	0.4903	0.4290	0.6824	0.4043	0.5663	0.3594	0.5982	0.3860	**0.3942**
(23,241;28,922]	0.5129	0.3228	0.6502	0.3788	0.3632	0.4223	0.7352	0.4199	0.6110	0.3696	0.6025	0.3890	**0.3987**
(28,922;38,218]	0.5317	0.3479	0.6170	0.3689	0.4540	0.4370	0.6911	0.4445	0.6044	0.3745	0.5903	0.4022	**0.4129**
(38,218;52,162]	0.4922	0.3589	0.6139	0.3825	0.5069	0.4516	0.6446	0.4622	0.6090	0.4553	0.5749	0.4208	**0.4305**
(52,162;74,886]	0.5368	0.3687	0.6224	0.3938	0.5772	0.4650	0.6092	0.4604	0.6168	0.4793	0.5896	0.4302	**0.4429**
(74,886;119,302]	0.4732	0.3635	0.6032	0.3927	0.5175	0.4553	0.5560	0.4553	0.5979	0.4613	0.5418	0.4237	**0.4338**
(119,302;247,282]	0.4177	0.3589	0.5970	0.3901	0.5448	0.4587	0.5795	0.4582	0.5969	0.3871	0.5359	0.4221	**0.4336**
247,282	0.4794	0.3336	0.5573	0.3929	0.5478	0.4140	0.5534	0.4180	0.5045	0.3432	0.5139	0.3908	**0.4148**
Total	**0.5007**	**0.3263**	**0.5998**	**0.3384**	**0.5208**	**0.4077**	**0.6423**	**0.4066**	**0.5669**	**0.3972**	**0.5666**	**0.3798**	**0.3846**
Total	**0.3333**		**0.3493**		**0.4095**		**0.4165**		**0.4146**		**0.3846**		**0.3846**

TABLE A.5 - *Partial coefficients of variation of recovery rates by total exposure, presence of either collateral or personal guarantee and borrower's residence area*

	North-West		North-East		Central Italy		South		Islands		Italy		Total
TE^C	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	
(0;547]	0.9354	1.7336	1.0690	1.6978	2.2360	1.4294	0.7500	1.4530	1.1859	1.3310	1.0134	1.5348	**1.5302**
(547;1,549]	0.9128	1.4466	0.8164	1.4547	0.3465	1.2727	1.1548	1.2826	0.6153	1.2161	0.6949	1.3424	**1.3380**
(1,549;2,066]	0.6543	1.4028	0.7937	1.3716	0.0000	1.1507	0.6577	1.1553	0.3998	1.2297	0.5703	1.2617	**1.2544**
(2,066;2,582]	0.8922	1.3405	0.5059	1.2919	0.8282	1.0698	0.8061	1.0769	0.4718	1.0540	0.7333	1.1702	**1.1653**
(2,582;3,615]	0.5725	1.2241	0.5766	1.1574	0.9386	0.9430	0.7935	0.9483	0.2760	0.8720	0.5980	1.0350	**1.0302**
(3,615;4,648]	0.7098	1.1387	0.5541	1.1192	0.5759	0.9093	0.4180	0.8386	0.3103	0.8249	0.4958	0.9636	**0.9582**
(4,648;5,681]	0.6210	1.0731	0.1578	1.0148	0.9798	0.8740	0.4993	0.8921	0.4663	0.7939	0.5578	0.9319	**0.9277**
(5,681;6,714]	0.5063	0.9950	0.7838	0.9929	0.7172	0.8142	0.5621	0.8001	0.4141	0.7887	0.5777	0.8743	**0.8705**
(6,714;8,263]	0.5899	0.9802	0.6452	0.9746	0.6230	0.7719	0.4651	0.8255	0.5156	0.7727	0.5333	0.8549	**0.8499**
(8,263;9,813]	0.5591	0.9334	0.6155	0.8789	0.6267	0.7689	0.5311	0.8045	0.4845	0.7733	0.5444	0.8259	**0.8223**
(9,813;11,362]	0.6975	0.9270	0.4801	0.9258	0.7045	0.7674	0.5457	0.7781	0.2931	0.7821	0.5528	0.8273	**0.8221**
(11,362;13,428]	0.9462	0.9301	0.5840	0.9544	0.5478	0.7133	0.5681	0.8069	0.4402	0.7534	0.5922	0.8160	**0.8118**
(13,428;16,010]	0.6846	0.9389	0.5256	0.8951	0.5990	0.7287	0.4160	0.7735	0.4214	0.7648	0.5069	0.8057	**0.7984**
(16,010;19,109]	0.6682	0.9357	0.5723	0.9163	0.5435	0.7098	0.3524	0.8265	0.4249	0.7793	0.5100	0.8142	**0.8048**
(19,109;23,241]	0.6938	0.9488	0.5055	0.9048	0.6659	0.6797	0.4286	0.7744	0.4332	0.7869	0.5229	0.7948	**0.7859**
(23,241;28,922]	0.6545	0.9749	0.4417	0.8767	0.7888	0.7054	0.3270	0.7497	0.3563	0.8195	0.4833	0.8041	**0.7901**
(28,922;38,218]	0.6011	0.9310	0.5540	0.9100	0.6648	0.6618	0.3441	0.7143	0.4002	0.8208	0.5048	0.7805	**0.7653**
(38,218;52,162]	0.6843	0.9008	0.5398	0.8672	0.5812	0.6264	0.4234	0.6616	0.4348	0.6927	0.5326	0.7355	**0.7236**
(52,162;74,886]	0.6004	0.8712	0.4902	0.8362	0.5577	0.6043	0.4384	0.6557	0.3888	0.6743	0.5005	0.7148	**0.6988**
(74,886;119,302]	0.6484	0.8575	0.4901	0.8098	0.5876	0.5924	0.4969	0.6374	0.4665	0.6809	0.5489	0.6986	**0.6867**
(119,302;247,282]	0.7668	0.8409	0.4692	0.7821	0.5340	0.5788	0.4654	0.6325	0.4116	0.8044	0.5477	0.6854	**0.6727**
>247,282	0.6479	0.8900	0.4997	0.7625	0.5639	0.6674	0.4725	0.6737	0.4412	0.7564	0.5071	0.7308	**0.6873**
Total	**0.6685**	**1.0509**	**0.5198**	**1.0145**	**0.6081**	**0.7795**	**0.4739**	**0.8652**	**0.4426**	**0.8635**	**0.5372**	**0.8918**	**0.8827**
Total	**1.0327**		**0.9906**		**0.7768**		**0.8478**		**0.8167**		**0.8827**		**0.8827**

TABLE A.6 - *Ratios of the partial average absolute deviations $S_{M_1}(r)$ and the corresponding average recovery rates by total exposure, presence of either collateral or personal guarantee and borrower's residence area*

	North-West		North-East		Central Italy		South		Islands		Italy		Total
TE^C	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	**secured**	**unsecured**	
(0;547]	0.8572	1.4942	1.0000	1.4269	1.6000	1.3414	0.6666	1.3570	1.1112	1.2773	1.0000	1.3944	**1.3921**
(547;1,549]	0.8000	1.2911	0.5000	1.2251	0.2667	1.1522	1.0000	1.1824	0.4763	1.1168	0.5973	1.2083	**1.2051**
(1,549;2,066]	0.5494	1.1974	0.6667	1.1407	0.0000	0.9795	0.5534	1.0612	0.3125	1.0967	0.4652	1.0943	**1.0889**
(2,066;2,582]	0.6667	1.1334	0.3509	1.0535	0.6499	0.9368	0.7693	0.9639	0.4022	0.9640	0.6839	1.0206	**1.0189**
(2,582;3,615]	0.4797	1.0372	0.4667	0.9841	0.7733	0.8286	0.7185	0.8492	0.2270	0.7737	0.5103	0.9059	**0.9034**
(3,615;4,648]	0.6154	0.9691	0.4273	0.9510	0.4702	0.7998	0.3187	0.7481	0.2378	0.7282	0.4163	0.8461	**0.8421**
(4,648;5,681]	0.5162	0.9238	0.1383	0.8873	0.7788	0.7715	0.3972	0.8041	0.3438	0.6992	0.4735	0.8236	**0.8209**
(5,681;6,714]	0.4374	0.8598	0.6900	0.8733	0.6645	0.7170	0.4589	0.7127	0.3590	0.6885	0.5066	0.7691	**0.7664**
(6,714;8,263]	0.4851	0.8514	0.5618	0.8396	0.4824	0.6774	0.3935	0.7394	0.4464	0.6742	0.4622	0.7509	**0.7466**
(8,263;9,813]	0.4881	0.8080	0.4816	0.7679	0.5829	0.6820	0.4178	0.7170	0.3865	0.6741	0.4686	0.7271	**0.7242**
(9,813;11,362]	0.6308	0.8072	0.4079	0.8164	0.5753	0.6802	0.4474	0.6925	0.2657	0.6881	0.4729	0.7319	**0.7272**
(11,362;13,428]	0.8229	0.8022	0.5056	0.8371	0.4395	0.6247	0.4730	0.7123	0.3576	0.6469	0.5125	0.7148	**0.7115**
(13,428;16,010]	0.6209	0.8101	0.4625	0.7901	0.5047	0.6328	0.3173	0.6839	0.3413	0.6560	0.4317	0.7032	**0.6977**
(16,010;19,109]	0.5847	0.8097	0.4785	0.8047	0.4400	0.6182	0.2681	0.7292	0.3419	0.6732	0.4240	0.7138	**0.7057**
(19,109;23,241]	0.6199	0.8196	0.4175	0.8016	0.5658	0.5860	0.3215	0.6822	0.3622	0.6661	0.4468	0.6938	**0.6862**
(23,241;28,922]	0.5594	0.8426	0.3530	0.7738	0.6313	0.6098	0.2481	0.6583	0.2961	0.7008	0.4070	0.7023	**0.6907**
(28,922;38,218]	0.5319	0.8066	0.5028	0.8008	0.5676	0.5682	0.2631	0.6254	0.3370	0.7009	0.4399	0.6818	**0.6687**
(38,218;52,162]	0.5959	0.7855	0.4646	0.7631	0.5029	0.5299	0.3418	0.5625	0.3688	0.5998	0.4589	0.6364	**0.6260**
(52,162;74,886]	0.5240	0.7578	0.4242	0.7374	0.4886	0.5067	0.3552	0.5584	0.3150	0.5800	0.4252	0.6160	**0.6010**
(74,886;119,302]	0.5505	0.7420	0.4062	0.7041	0.4987	0.4915	0.4081	0.5407	0.4002	0.5808	0.4599	0.5959	**0.5855**
(119,302;247,282]	0.6641	0.7239	0.4040	0.6739	0.4473	0.4785	0.3869	0.5297	0.3416	0.6905	0.4667	0.5826	**0.5715**
>247,282	0.5423	0.7527	0.4127	0.6607	0.4894	0.5568	0.3995	0.5687	0.3600	0.6241	0.4174	0.6192	**0.5788**
Total	**0.5838**	**0.9090**	**0.4446**	**0.8868**	**0.5292**	**0.6772**	**0.3898**	**0.7683**	**0.3667**	**0.7540**	**0.4569**	**0.7823**	**0.7748**
Total	**0.8962**		**0.8697**		**0.6747**		**0.7539**		**0.7123**		**0.7748**		**0.7748**