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MULTIVARIATE NONPARAMETRIC TESTING FOR COMPARING SECTOR CREDIT RISK

Marco Marozzi*
Luigi Santamaria**

SUMMARY

After the Basel II accord, banks should not distribute funds without considering which sector firms belong to, as usually done. In this context, we would like to compare firm sector for what concern different financial ratios. Through a suitable multivariate nonparametric test we evaluate whether or not sectors can be distinguished with respect to the ratios. The analysis of a data set about positions from a medium size Italian financial institution clearly contradict the common banking practice of distributing funds without considering which sector firms belong to and strongly recommend for alternative credit treatment.

Keywords: *Nonparametric Testing, Multivariate Testing, Multiple Comparisons, Credit Risk, Financial Indicators.*

1. INTRODUCTION

On July 4, 2006, the Basel Committee on Banking Supervision released a comprehensive version of the Basel II Accord: International Convergence of Capital Measurement and Capital Standards. Basel II is the second of the Basel Accords, which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. The aim of these recommendations is to ensure that capital allocation is more risk sensitive, to separate operational risk from credit risk, to align economic and regulatory capital more closely to reduce the scope for regulatory arbitrage. Basel II uses a "three pillars" concept. The first pillar deals with maintenance of regulatory capital. The second pillar deals with the regulatory response to the first pillar, giving regulators much improved tools over those available to them before. The third pillar greatly increases the disclosures that the bank must make and is designed to allow the market to have a better picture of the overall risk position of the bank and to allow the counterparties of the bank to price and deal appropriately.

* Dipartimento di Economia e Statistica - Università della Calabria - via P. Bucci, Cubo 0c, 87036 RENDE (CS) (e-mail: marozzi@unical.it). Corresponding author.

** Dipartimento di Scienze Statistiche - Università Cattolica del Sacro Cuore - Largo A. Gemelli, 1, 20123 MILANO (e-mail: luigi.santamaria@unicatt.it).

The paper has been written by M. Marozzi. L. Santamaria gave very helpful indications for preparing Section 1 and Section 4 and for presenting the results in Section 3.

TABLE 1. - *Sector classification*

Sector	Bank of Italy Code
"Società produttive"	430
"Quasi società artigiane"	48
- 20 or more employees	*480*
- more than 5 and less than 20	*481*
- 5 or less employees	*482*
"Quasi società non artigiane"	49
- 20 or more employees	*490*
- more than 5 and less than 20	*491*
- 5 or less employees	*492*
"Famiglie produttrici artigiane"	614
"Famiglie produttrici non artigiane"	615

A central aspect of Basel II is the use of credit risk models in calculating the regulatory capital. In particular, institutions are allowed to develop their own proprietary credit models in order to provide internal ratings and evaluate the expected default frequencies related to their credit facilities. One of the possible effects of an unselective use of these models would be a less favourable treatment of small businesses and middle market borrowers, which are significant in Italian corporate positions. For this reason, credit risk management should be referred to homogeneous positions in relation to their specific economic and financial structure. In particular, a set of five balance sheet ratios should be considered

- the return on equity ROE, which is equal to a fiscal year's net income divided by total equity. It measures a firm's efficiency at generating profits from net assets, and shows how well a company uses investment to generate earning growth;
- the return on assets ROA, which is equal to earning before interest and taxes (EBIT) divided by total assets. It indicates how much earning derive from assets (capital intensity);
- the debt to equity ratio D/E, that indicates the relative proportion of debt and equity used to finance a company's assets;
- the current ratio CR, which is equal to total current assets divided by total current liabilities. It indicates the company's ability to meet short-term debt obligations; the higher the ratio, the more liquid the company is;
- the ratio of the net income before extraordinary items to revenues.

For more details on the ratios see e.g. Rees (1995). A dataset of these ratios about positions from a medium size Italian financial institution have been analysed. Table 1 shows the firm classification into nine sectors following Bank of Italy watch committee, see for more details www.Bancaditalia.it\Vigilanza_tutela. Our aim is not to distinguish good and bad firms (a distinction usually obtained via discriminant analysis) but to evaluate, through a suitable multivariate nonparametric test, if sectors can or cannot be distinguished with respect to the indicators.

2. NONPARAMETRIC TESTING FOR CREDIT RISK COMPARISON

The problem under issue, can be stated as a multivariate two-sample location problem with multiple comparisons. We address this problem within the nonparametric permutation framework because of the various advantages of permutation tests with respect to parametric tests (Marozzi, 2002). A permutation test is a statistical procedure for hypothesis testing in which one calculates the value the test statistic assumes on the observed data and on all permutations of the data (or on a large random sample of them) to decide whether to accept or reject the null hypothesis. More precisely, the p-value is computed as the proportion of permutations that have test statistic greater than or equal to the observed one.

The main characteristic of permutation tests lies in their adaptability to many different applications. They may be applied to continuous, ordered or categorical, normally or non-normally distributed data, to homogeneous or heterogeneous data, in both univariate and multivariate fields, to single or to repeated measurements. Pesarin (2001) underlines that there exist problems that can be addressed only within a permutation framework. These include, among others, problems on multivariate categorical variables, data missing either completely at random or not, multidimensional analysis of variance with variables partly quantitative and partly categorical, situations in which the number of units is less than the number of observed variables (as often happens with repeated measurements). He also outlines a sort of Bayesian way to permutation testing. Regarding problems that have already been solved within a parametric framework, it is very often possible to develop the permutation version of the suggested test statistic. The performance of this latter is generally similar to that of the parametric test when the assumptions behind this one are met, otherwise its performance could be better (see Good, 2000, and the references therein). The permutation framework allows us to propose ad hoc statistics for the particular problem we are dealing with. We are not forced to use either the permutation version of statistics based on the classical theory or to examine usual alternative hypotheses.

Another quality of permutation tests is their robustness. A parametric test is exact only if the underlying distribution is that on which the test is based. In this case, the test is often the best available test. If the underlying distribution is different, the parametric test loses these properties and its performance may be poor. On the contrary, provided that the observations are exchangeable under the null hypothesis, a permutation test is always exact and unbiased against shifts in the direction of higher values (Good, 2000).

Some authors claim that permutation tests have many similarities with bootstrap tests, underlining that both procedures employ only the data at hand to draw inference on the null hypothesis, require minimal assumptions and are computer-intensive. However, according to other authors, there are few similarities between these methods, because permutation tests refer to the context of conditional inference, while bootstrap tests do not, and ensure asymptotical properties only. According to results due principally to Romano (1989), in many common situations the permutation and the bootstrap distributions of a certain test statistic are asymptotically the same. Bootstrap tests have an advantage over permutation tests: they can also be used to test null hypotheses other than that of invariance, whereas permutation tests may be used only to test null hypotheses of invariance with respect to the permutation model underlying the problem. However, permutation tests are generally preferred because they are exact even for small samples and, being conditional on a set of sufficient statistics, they have properties which are not satisfied by bootstrap tests. In fact, bootstrap tests are, at least for small samples, neither exact nor conservative, since the probability of a type-one error may be greater than the nominal significance level.

For tackling the sector comparison problem we follow the Nonparametric Combination of Dependent Tests theory due to Pesarin (2001). At the basis of the theory there is a natural idea, that of breaking down a (complex) problem into a set of easier to solve sub-problems, each of which is related to a particular aspect of the original problem. In a typical complex problem, the dataset consists of $C \geq 2$ independent random samples in which the values of $q \geq 1$ random variables, generally not independent, are observed. Suppose that the null hypothesis is that the C samples come from the same population $H_0 : F_1 = F_2 = \ldots = F_C$, where F_l denotes the distribution function of the lth population ($l = 1, \ldots, C$) against $H_1 : F_h \neq F_l$ for some pair (h, l). Suppose that m partial aspects may be emphasized, so that H_0 may be broken down into m sub-hypotheses as $H_0 = \bigcap_{j=1}^{m} H_{0j}$, that is to say that H_0 is true if all the H_{0j} are jointly true. Note that $H_1 = \overline{H}_0 = \overline{\bigcap_{j=1}^{m} H_{0j}} = \bigcup_{j=1}^{m} \overline{H}_{0j}$ and that for most multivariate problems $m = q$ and $H_0 = \bigcap_{j=1}^{m} H_{0j} = \bigcap_{j=1}^{q} \left(F_{1j} = F_{2j} = \ldots = F_{Cj}\right)$, where F_{ij} denotes the distribution function of the jth random variable in the population underlying the ith sample. To solve the problem, as a first step, we may test each of the m partial null hypotheses. We assume to have an unbiased and consistent permutation test. Without loss of generality, this test is assumed to be significant for large values of the test statistic. These assumptions are rather general because it is usually not difficult to find tests which satisfy them. It follows that the permutation distribution functions of the partial test statistics are stochastically larger under the alternative hypothesis H_{1j} than under H_{0j} and so the associated p-values are positively dependent. After accomplishing the first step, by computing the values of the m partial test statistics, one can perform the final step, by combining the partial p-values, which are permutationally equivalent to test statistics values, through an appropriate function in order to test the global null hypothesis.

After pointing out that the considered global null hypothesis implies the exchangeability of the observations and then enables to use a permutation procedure, it is worth emphasizing that the major feature of the Nonparametric Combination of Dependent Tests is that one is not required to specify the dependence structure of partial tests. Since the underlying dependence relation structure is nonparametrically captured by the combining procedure, one may pay attention to the set of partial tests. Therefore the researcher has only to make sure that the partial tests satisfy the rather general assumptions which we reported earlier. If these assumptions are satisfied, then the combined test is exact, unbiased and usually with good power behaviour.

In the application at issue, the (global) null hypothesis is that two sectors cannot be distinguished for what concern the five financial indicators previously outlined. Let **X** denote the data matrix

$$\mathbf{X} = \begin{bmatrix} X_{111} & \dots & X_{1n_11} & X_{211} & \dots & X_{2n_21} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{11j} & \dots & X_{1n_1j} & X_{21j} & \dots & X_{2n_2j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{11J} & \dots & X_{1n_1J} & X_{21J} & \dots & X_{2n_2J} \end{bmatrix} =$$

$$= \begin{bmatrix} X_{11} & \dots & X_{n_11} & X_{(n_1+1)1} & \dots & X_{(n_1+n_2)1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{1j} & \dots & X_{n_1j} & X_{(n_1+1)j} & \dots & X_{(n_1+n_2)j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{1J} & \dots & X_{n_1J} & X_{(n_1+1)J} & \dots & X_{(n_1+n_2)J} \end{bmatrix} = \begin{bmatrix} \underline{X}_1 \\ \vdots \\ \underline{X}_j \\ \vdots \\ \underline{X}_J \end{bmatrix}$$

where X_{i1j} (X_{i2j}) is the ith observation on the jth variable for the first (second) sector. We formalize the hypothesis through the means of the five variables (i.e. financial indicators) $X_j, j = 1, ..., 5$

$$H_0 = \bigcap_{j=1}^{5} H_{0j}, \text{ where } H_{0j} : \delta_{1j} = \delta_{2j}$$

and where $\delta_1 j$ $(\delta_2 j)$ is the mean of the jth variable in the first (second) sector under comparison. The alternative hypothesis is

$$H_1 = \bigcup_{j=1}^{5} H_{1j}, \text{ where } H_{1j} : \delta_{1j} \neq \delta_{2j}.$$

To test the five systems of partial hypotheses we use the statistic

$$T_j^* = \left| \frac{1}{n_1} \sum_{i=1}^{n_1} X_{1ij}^* - \frac{1}{n_2} \sum_{i=1}^{n_2} X_{2ij}^* \right| = \left| \frac{1}{n_1} \sum_{i=1}^{n_1} X_{ij}^* - \frac{1}{n_2} \sum_{i=n_1+1}^{n_1+n_2} X_{ij}^* \right|$$

where X_{1ij}^* is the ith element of a permutation $\underline{X}_j^*$ of the jth pooled sample

$$\underline{X}_j^* = \left(X_{11j}^*, ..., X_{1n_1j}^*, X_{21j}^*, ..., X_{2n_2j}^* \right) = \left(X_{1j}^*, ..., X_{n_1j}^*, X_{(n_1+1)j}^*, ..., X_{(n_1+n_2)j}^* \right).$$

The observed value of T_j^* is ${}_0T_j = \left| \frac{1}{n_1} \sum_{i=1}^{n_1} X_{1ij} - \frac{1}{n_2} \sum_{i=1}^{n_2} X_{2ij} \right|$. Note that we intrinsically assume homoschedasticity but we do not assume anything else about populations. The p-value of the test is estimated by

$$\hat{L}_{T_j}\left({}_0T_j\right) = \frac{1}{K+1} \left[\sum_{k=1}^{K} I\left({}_kT_j^* \geq {}_0T_j \right) + 0.5 \right],$$

where ${}_kT_j^*$ is the kth permutation value of T_j^*. With respect to the proportion of permutations that have test statistic greater than or equal to the observed one, 0.5 and 1 have been added respectively to the numerator and denominator of the fraction. The reason for this modification is to obtain estimated p-values in the open interval]0,1[avoiding computational problems which may arise in the second step of the procedure (e.g. ln(0)). Since large K are used, the correction is practically irrelevant. For more details on the practical irrelevancy of the modification see Pesarin (2001, p. 144). Efficient algorithms to compute exact permutation p-values for some univariate problems are available (see e.g. Mehta *et al.* 1988 and Marozzi, 2004). Such algorithms compute exactly the permutation test statistic distribution in the tails, but to perform a combined test the whole distribution should be studied. Since algorithms for exact multivariate p-value computing are not available in the literature we have to estimate them. It should be noted that the T_j test is exact, unbiased and consistent (Pesarin, 2001).

The second step of the procedure deals with the nonparametric combination of $T_1, ..., T_J$ tests. In particular, the test statistic for the multivariate problem is obtained by nonparametric combination of the p-values associated with the partial tests. Let φ be a proper combining function, the test statistic is

$$^{\varphi}T^* = \varphi(L_{T_1}, ..., L_{T_J}).$$

The observed value of $^{\varphi}T^*$ is estimated as ${}^{\varphi}_{0}T = \varphi(L_{T_1}({}_0T_1), ..., L_{T_J}({}_0T_J))$, and its distribution is simulated by means of the same K permutations of the first step. For example, the kth permutation value of $^{\varphi}T^*$ is computed as ${}^{\varphi}_{k}T^* = \varphi\left(L_{T_1}\left({}_kT_1^*\right), ..., L_{T_J}\left({}_kT_J^*\right)\right)$. Large values of the observed test statistic are evidence against the null hypothesis. The p-value of the test is estimated by $L_{^{\varphi}T^*} = \sum_{k=1}^{K} I\left({}^{\varphi}_{k}T^* \geq {}^{\varphi}_{0}T\right) \Big/ K$ and H_0 is rejected if $L_{^{\varphi}T^*} \leq \alpha$. The most often used combining functions are

(i) the Fisher combining function $\varphi_F = \sum_{j=1}^{J} \ln\left(1/L_{T_j}\right)$;

(ii) the normal (Liptak) combining function $\varphi_N = \sum_{j=1}^{J} \Phi^{-1}\left(1 - L_{T_j}\right)$, where Φ denotes the standard normal distribution function;

(iii) the Tippett combining function $\varphi_T = \max(1 - L_{T_1}, ..., 1 - L_{T_J})$.

There exist situations in which one combining function is preferable to the others (Pesarin, 2001). For example, the Tippett combining function is suggested when only one (or a few) among H_{1j}s is expected to be true and that of Liptak when H_{1j}s are expected to be jointly true. Since the Fisher combining function has an intermediate behavior, it is suggested when nothing is expected about partial alternative hypotheses, and for this reason we adopt it for the multivariate test. This corresponds to take as observed value of the multivariate test statistic

$${}_0T = -\sum_{j=1}^{J} \ln\left(L_{T_j}\left({}_0T_j\right)\right),$$

and as the p-value $L_T = \sum_{k=1}^{K} I({}_kT^* \geq {}_0T) \Big/ K$, where ${}_kT^* = -\sum_{j=1}^{J} \ln \left(L_{T_j}\left({}_kT_j^* \right) \right)$. H_0 is rejected if $L_T \leq \alpha$. The T test is exact, unbiased and consistent (Pesarin, 2001).

We underline that rejecting H_0 means that the data at hand suggest that the two sector are different for what concern the five financial indicators. It should be emphasized that the researcher is not required to specify the dependence structure of partial tests and variables because the underlying dependence structure is nonparametrically captured by the combining procedure. A two level inference may be drawn: the first level is the partial one and it is on each single variable, the second one is the global one (which is that of interest, here) and allows us to summarize the multivariate comparison through only one indicator. In presenting the results, the multiple comparison problem has been taken into account via the Bonferroni's correction (where the corrected p-value is the raw p-value multiplied by the number of comparisons).

3. APPLICATION AND DISCUSSION

We applied the method presented in the previous section to the data set presented in the first one. This section present the results. We consider different classifications for the firms. Table 2 displays the results about the standard classification. The Bonferroni corrected p-value relative to the multivariate comparison between row and column sectors are reported in the inner cells. 10,000 permutations have been computed. p-values less then 0.05 indicate different sector positions with respect to the considered ratios.

TABLE 2. - *Corrected p-values: standard classification*

sector	480-1 490-1	482 492	614	615
430	0.0120	< 0.0001	< 0.0001	< 0.0001
480-1 490-1		0.4950	0.4550	0.8082
482 492			1	1
614				1

As it can be seen, since the corresponding p-values are less than 0.05, the 430 sector (''società produttive'') is significantly different with respect to the others that are: 480-481-490-491 (''quasi società artigiane'' and ''non artigiane'' with more than 5 employees); 482-492 (''quasi società artigiane'' and ''non artigiane'' with 5 or less employees); 614 (''famiglie produttrici artigiane'') and 615 (''famiglie produttrici non artigiane''). These results speak against the common banking practice of distributing funds without considering which sector firms belong to and strongly recommend for alternative credit treatment. What happens if we consider a more or less detailed firm classification? Table 3 reports the results for a more detailed sector classification, and Table 4 for a less detailed classification. The tests which correspond to the comparison between the 430 sector (“società produttive”) and the remaining are always significant at the 5% level which means that the 430 sector is different with respect to the other ones as far as the return on equity, the return on assets, the debt to equity ratio, the current ratio and the ratio of the net income before extraordinary items to revenues are concerned.

Nevertheless, it should be cautioned that, while the underlying results seem to indicate disparity between sectors, the proposed method should seek a prudential logic combined with qualitative information.

TABLE 3. - *Corrected p-values: a more detailed sector classification*

sector	480-1	482	490-1	492	614	615
430	0.0336	< 0.0001	0.0137	0.0021	< 0.0001	< 0.0001
480-1		1	1	1	1	1
482			1	1	1	1
490-1				1	0.5523	0.8799
492					1	1
614						1

TABLE 4. - *Corrected p-values: a less detailed sector classification*

<table>
<tr><th>sector</th><th>480-1</th><th>482</th><th>490-1</th></tr>
<tr><td>430</td><td>0.0072</td><td>< 0.0001</td><td>< 0.0001</td></tr>
<tr><td>480-1</td><td rowspan="2"></td><td rowspan="2">0.2970</td><td rowspan="2">0.2598</td></tr>
<tr><td>490-1</td></tr>
<tr><td>482</td><td rowspan="2" colspan="2"></td><td rowspan="2">1</td></tr>
<tr><td>492</td></tr>
</table>

Moreover, the proposed method is only based on “*ex post*” ratios. This was done in accordance with the standard banking practice, whereas it may be useful to take also into account “*ex ante*” information. In fact, banks distribute funds in order to support the development of a firm, but to decide how many funds should be distributed, they generally take into account only ex-post budget information and not perspective indicators (Santamaria, 2003).

4. CONCLUSION

Basel II is the second of the Basel Accords, which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. A central aspect of Basel II is the use of credit risk models in calculating the regulatory capital. One of the possible effects of an unselective use of these models would be a less favourable treatment of small businesses and middle market borrowers, which are significant in Italian corporate positions. After the Basel II accord, banks should not distribute funds without considering which sector firms belong to, as usually done. The analysis of a data set about positions from a medium size Italian financial institution clearly contradict the common banking practice of distributing funds without considering which sector firms belong to and strongly recommend for alternative credit treatment. The data have been analysed through a multivariate nonparametric test. The test has been developed within the nonparametric combination methodology and it is exact, unbiased and consistent.

RIASSUNTO

Con l'adozione dei criteri di Basilea 2, gli istituti finanziari non potranno più procedere con l'erogazione di finanziamenti senza fare (com'è prassi) distinzioni fra i settori di appartenenza. In questo contesto, ci si propone di confrontare tra loro diversi settori di impresa con riferimento ad alcuni indicatori finanziari di bilancio. Tramite un opportuno test nonparametrico multivariato si vuole valutare se i settori siano o meno distinguibili rispetto agli indicatori di bilancio considerati. I risultati dell'analisi fatta su un insieme di dati forniti da un istituto finanziario di medie dimensioni suggerisce in maniera chiara come il Sistema Bancario dovrebbe abbandonare la prassi di una politica del credito indifferenziata per settore, a favore di una politica che tenga conto in maniera esplicita delle categorie aziendali.

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