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COMPARISON OF SEVERAL CONFIDENCE INTERVALS FOR NORMAL DISTRIBUTION WITH KNOWN COEFFICIENT OF VARIATION

M.T. Alodat*
Z.R. Al-Rawi*
W.A. Abu-dayyeh**

SUMMARY

In this paper, we consider the problem of constructing confidence intervals for the normal population mean when the coefficient of variation is known. We obtain 13 confidence intervals using different pivots. Also we conduct a simulation study to compare the expectations as well as the standard deviations of the intervals lengths.

Keywords: *Coefficient of Variation, Normal Distribution, Pivotal Method, Confidence Interval.*

1. INTRODUCTION

In many fields of science like agriculture, biology and sample surveys, it could be possible to have a statistical population with known coefficient of variation (c.v.). Several statisticians have considered the estimation of distribution mean when the population c.v. is known (Gleaser and Hearly, 1976; Wencheko and Wijekoon, 2005; Laheetharan and Wijekoon, 2007). Marshall (1936), has studied the frequency distribution of the insect population under the assumption that the population c.v. is known. Based on the birds survey for six provinces in Canada during the period 1967-1970, Sen (1978) showed that the c.v. of kill/hunter for each province remains stable over the period. Sen has obtained an estimate for the population mean when the population c.v. is known. Assuming the c.v. is known, Searls (1964, 1967) obtained an estimate for the population mean which is more efficient than the sample mean. Sinha (1983) obtained the maximum likelihood and the Bayes' estimates of the normal population mean with positive mean and known c.v. Without loss of generality, he assumed c.v.=1. Also he conducted a simulation study to compare the two estimators. Singh and Mathur (2005) have suggested estimators for the population mean in the presence of scrambled responses. They have treated two cases: (i) the c.v. of the response variable known in absence of errors, and (ii) the guessed value of the c.v. of response variable is available. Also the suggested estimators have been examined numerically.

* Department of Statistics - Yarmouk University Irbid - JORDAN (e-mail: alodatmts@yahoo.com; alrawiz@yu.edu.jo).
** Department of Mathematics and Statsitics - Sultan Qabous University - OMAN (e-mail: abudayyehw@yahoo.com).

It can be noted, as in the above review, that the problem of making statistical inference about a population with known c.v. has significant applications in many fields of science. Moreover, constructing confidence intervals using pivotal method to the normal population with known c.v. has not been discussed extensively in the literature. For this, we consider the problem of constructing confidence intervals for the normal population mean using the pivotal method.

The following notation will be used in the sequel. We use $X_{(1)}, \dots, X_{(n)}$ to denote the order statistics of a random sample $X_1, \dots, X_n$ from $N(\mu, \sigma^2)$, and $Z_{(1)}, \dots, Z_{(n)}$ the order statistics of the corresponding z-scores. The Student and Chi-square random variables with n degrees of freedom are denoted by t_n and χ^2_n, respectively. Also their 100α quantiles are denoted by $t_{n,\alpha}$ and $\chi^2_{n,\alpha}$, respectively. The density and the distribution functions of $N(0,1)$ are, respectively, denoted by $\phi(x)$ and $\Phi(x)$. Also, let $Z_{1-\alpha/2}$ is the 100(1-$\alpha/2$) quantile of $N(0,1)$, $t_{n-1,1-\alpha/2}$ is the 100(1-$\alpha/2$) quantile of student distribution with $n-1$ degrees of freedom and $\chi^2_{n-1,\ 1-\alpha/2}$ and $\chi^2_{n-1,\ \alpha/2}$ are the $100\left(\dfrac{\alpha}{2}\right)$ and $100\left(1-\dfrac{\alpha}{2}\right)$ quantiles of the chi-square distribution with $n-1$ degrees of freedom, respectively.

In this paper, we construct 13 confidence intervals for the parameter θ based on a random sample from $N(\theta,\ \theta^2)$. Let $X_1, \dots, X_n$ be a random sample from $N(\theta,\ \theta^2)$ and $\bar{X}$ and S^2 denote its sample mean and sample variance, respectively. We assume that the sign of θ is known. Without loss of generality, assume $\theta > 0$. Constructing confidence intervals as well as prediction intervals, based on statistics, has a long history in literatures. For example, the sample range has been widely used in statistical inference especially in industrial quality control. Hartley (1950) has analyzed the randomized block design by ranges. The use of ranges in his analysis was motivated by the high cost of sampling units. Also Jong-Wuu, Wen-Chuan and Sheau-Chiann (2007) proposed several pivotal quantities to find several classical prediction intervals for a future ordered statistic based on the difference of two ordered statistics of a sample. Zhange and Al-Saleh (2003) have improved the best linear unbiased estimator of the scale parameter of symmetric distributions using the absolute value of ranked set sample. Their estimator can be easily employed to find a pivot and then to construct a confidence interval. For literature about this topic see (Hartley, 1950; Jong-Wuu *et al.*, 2007; Jong-Wuu, Wen-Chuan and Sheau-Chiann, 2005; Jamalizadeh and Balkrishnan, 2009).

In many applications, it is difficult or expensive to quantify sampling units, but they can be ranked easily, using a visual inspection without quantifying them actually (Zhange and Al-Saleh, 2003). In this case, we use some visual inspection or any other cheap method to detect the order statistics of a random sample and then we consider them for actual quantification. This process produces a set of order statistics which constitute sufficient statistics to the problem of interest. Under situation, we may rely on order statistics to draw inference. In the other hand, some estimators of θ based on the sample mean may lead to inadmissible estimators, since $\theta > 0$, while the sample mean assumes negative values with positive probability. For this reason, we consider pivots which are functions order statistics or the absolute values of the sample values. In this paper, we consider the 13 pivots to construct $100(1-\alpha)\%$ C.I.'s for θ. We compare these interval via the expected and standard deviation of their lengths.

2. PIVOTS

Pivot 1. $P_1 = \sqrt{n}\left(\frac{\bar{X}-\theta}{\theta}\right) \sim N(0,\ 1)$. Based on this pivot, a 100(1 - α)% C.I. for θ is given by

$$\left(\frac{\bar{X}}{1+\frac{Z_{1-\alpha/2}}{\sqrt{n}}}, \frac{\bar{X}}{1-\frac{Z_{1-\alpha/2}}{\sqrt{n}}}\right) \tag{1}$$

Its expected length is

$$E(L_1) = \frac{2\theta Z_{1-\alpha/2}}{\sqrt{n}\left(1-\frac{Z^2_{1-\alpha/2}}{n}\right)}.$$

Straight forward calculations show that the standard deviation of L_1 is

$$\sigma_{L_1} = \frac{2\theta Z_{1-\alpha/2}}{n\left(1-\frac{Z^2_{1-\alpha/2}}{n}\right)}.$$

Pivot 2. $P_2 = \frac{\sqrt{n}(\bar{X}-\theta)}{S} \sim t_{n-1}$.A 100(1 - α)% C.I. for θ is given by

$$\left(\bar{X} - t_{n-1,1-\alpha/2}\frac{S}{\sqrt{n}}, \bar{X} + t_{n-1,1-\alpha/2}\frac{S}{\sqrt{n}}\right), \tag{2}$$

where the expected length of this interval is

$$E(L_2) = \frac{2\ t_{n-1,1-\alpha/2}}{\sqrt{n}}E(S).$$

We need to obtain the value of $E(S)$ which can be found as follows. Let $Y = \frac{(n-1)S^2}{\theta^2}$.

From normal distribution theory, we have

$$Y = \frac{(n-1)S^2}{\theta^2} \sim \chi^2_{n-1}.$$

Since $S = \frac{\theta\sqrt{Y}}{\sqrt{n-1}}$, then $E(S) = \frac{\theta}{\sqrt{n-1}}E(Y^{\frac{1}{2}}) = \frac{\theta}{\sqrt{n-1}}\frac{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$. So, the mean value of L_2 is

$$E(L_2) = \frac{2^{\frac{3}{2}}\ \theta\ t_{n-1,1-\alpha/2}}{\sqrt{n(n-1)}} \cdot \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}.$$

Since $L_2^2 = \frac{4}{n} t^2_{n-1,\ 1-\alpha/2} S^2$, then the variance of L_2 is

$$\sigma^2_{L_2} = E(L_2^2) - [E(L_2)]^2.$$

Since $E(L_2^2) = \frac{4}{n} t^2_{n-1,\ 1-\alpha/2} \theta^2$, then the standard deviation of L_2 is

$$\sigma_{L_2} = \theta \sqrt{\frac{4t^2_{n-1,1-\alpha/2}}{n(n-1)} \left[(n-1) - \left(\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \right)^2 \right]}.$$

Pivot 3. $P_3 = \frac{(n-1)S^2}{\theta^2} \sim \chi^2_{(n-1)}$. Based on P_3, a 100(1-α)% C.I. for θ is

$$\left(\frac{\sqrt{n-1}S}{\sqrt{\chi^2_{n-1,1-\alpha/2}}}, \frac{\sqrt{n-1}S}{\sqrt{\chi^2_{n-1,\alpha/2}}} \right). \tag{3}$$

The interval (3) has an expected length of

$$E(L_3) = \sqrt{n-1}\left(\frac{b-a}{ab}\right) E(S),$$

$$= \left(\frac{b-a}{ab}\right)\theta \frac{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)},$$

where $a = \sqrt{\chi^2_{n-1,\ 1-\alpha/2}}$ and $b = \sqrt{\chi^2_{n-1,\ \alpha/2}}$.

The variance of L_3 is

$$\sigma^2_{L_3} = (E(L_3))^2 - (EL_3)^2$$

$$= (n-1)\left(\frac{b-a}{ab}\right)^2 \theta^2 - \left(\frac{b-a}{ab}\right)^2 \theta^2 \left(\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \right)^2$$

$$= \left(\frac{b-a}{ab}\right)^2 \theta^2 \left[(n-1) - \left(\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \right)^2 \right].$$

The standard deviation of $L_3$is

$$\sigma_{L_3} = \theta\sqrt{n-1}\left(\frac{b-a}{ab}\right)\left[1 - \frac{\Gamma\left(\frac{n}{2}\right)^2}{(n-1)\Gamma\left(\frac{n-1}{2}\right)^2}\right]^{1/2}.$$

Pivot 4. $P_4 = \frac{|X_{(n)}|}{\theta} = |1 + Z_{(n)}|.$

Let $a_{4,\,\alpha/2}$ and $b_{4,\,1-\alpha/2}$ be the $100\left(\frac{\alpha}{2}\right)$ and $100\left(1-\frac{\alpha}{2}\right)$quantiles of P_4. Now a 100(1 - α) % C.I. for θ is

$$\left[\frac{|X_{(n)}|}{b_{4,\,1-\alpha/2}}, \frac{|X_{(n)}|}{a_{4,\,\alpha/2}}\right].$$

Its length has mean and standard deviation given by

$$E(L_4) = E(|X_{(n)}|)\left(\frac{1}{a_{4,\,\alpha/2}} - \frac{1}{b_{4,1-\alpha/2}}\right)$$

and

$$\sigma_{L_4} = Var(|X_{(n)}|)\left(\frac{1}{a_{4,\,\alpha/2}} - \frac{1}{b_{4,1-\alpha/2}}\right)^2$$

Pivot 5. $P_5 = \frac{|X_{(1)}|}{\theta} = |1 + Z_{(1)}|.$

Let $a_{5,\,\alpha/2}$ and $b_{5,\,1-\alpha/2}$ be the $100\left(\frac{\alpha}{2}\right)$ and $0\left(1-\frac{\alpha}{2}\right)$ quantiles of P_5. Now a $100(1-\alpha)\%$ C.I. for θ is

$$\left[\frac{|X_{(1)}|}{b_{5,\,1-\alpha/2}}, \frac{|X_{(1)}|}{a_{5,\,\alpha/2}}\right].$$

It has the, respectively the length mean and length standard deviation

$$E(L_5) = E(|X_{(1)}|)\left(\frac{1}{a_{5,\,\alpha/2}} - \frac{1}{b_{5,1-\alpha/2}}\right)$$

and

$$\sigma_{L_5} = Var(|X_{(1)}|)\left(\frac{1}{a_{5,\,\alpha/2}} - \frac{1}{b_{5,1-\alpha/2}}\right)^2.$$

Pivot 6.
$$\begin{cases} \dfrac{|x_{(m)}|}{\theta} = |1 + Z_{(m)}|, & n = m - 1, \\ \dfrac{|x_{(m)} + X_{(m+1)}|}{2\theta} = |2 + Z_{(m)} + Z_{(m+1)}| & n = 2m. \end{cases}$$

A $100(1-\alpha)\%$ C.I. for θ is

$$\begin{cases} \left(\dfrac{|X_{(m)}|}{b_{6,\,1-\frac{\alpha}{2}}}, \dfrac{|X_{(m)}|}{a_{6,\,1-\frac{\alpha}{2}}} \right), & n = 2m - 1, \\ \left(\dfrac{|X_{(m)} + X_{(m+1)}|}{2b_{6,\,1-\frac{\alpha}{2}}}, \dfrac{|X_{(m)} + X_{(m+1)}|}{2a_{6,\,1-\frac{\alpha}{2}}} \right) & n = 2m. \end{cases}$$

Pivot 7. $P_7 = \dfrac{x_{(n)} - x_{(1)}}{\theta} = Z_{(n)} - Z_{(1)}.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{X_{(n)} - X_{(1)}}{b_{7,\,1-\frac{\alpha}{2}}}, \frac{X_{(n)} - X_{(1)}}{a_{7,\,\frac{\alpha}{2}}} \right).$$

Pivot 8. $P_8 = \dfrac{|x_{(n)} + x_{(1)}|}{\theta} = |2 + Z_{(n)} + Z_{(1)}|.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{|X_{(n)} + X_{(1)}|}{b_{8,\,1-\frac{\alpha}{2}}}, \frac{|X_{(n)} + X_{(1)}}{a_{8,\,\frac{\alpha}{2}}} \right).$$

Pivot 9. $P_9 = \dfrac{x_{(n)} - x_{(m)}}{\theta} = Z_{(n)} - Z_{(m)}.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{X_{(n)} - X_{(m)}}{b_{9,\,1-\frac{\alpha}{2}}}, \frac{X_{(n)} - X_{(m)}}{a_{9,\,\frac{\alpha}{2}}} \right).$$

Pivot 10. $P_{10} = \dfrac{|x_{(1)}x_{(n)}|}{\theta^2} = |(1 + Z_{(1)})(1 + Z_{(n)}|.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{\sqrt{|X_{(1)}X_{(n)}|}}{\sqrt{b_{10,\,1-\frac{\alpha}{2}}}}, \frac{\sqrt{|X_{(1)} + X_{(n)}}}{\sqrt{a_{10,\,\frac{\alpha}{2}}}} \right).$$

Pivot 11. $P_{11} = \dfrac{|x_1 x_2 \cdots x_n|}{\theta^n} = |(1+Z_1)(1+Z_2)\cdots(1+Z_n)|.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{|X_1 X_2 \cdots X_n|^{\frac{1}{n}}}{\left(b_{11,\,1-\frac{\alpha}{2}}\right)^{1/n}}, \frac{|X_1 X_2 \cdots X_n|^{\frac{1}{n}}}{\left(a_{11,\,\frac{\alpha}{2}}\right)^{1/n}} \right).$$

Pivot 12. $P_{12} = \dfrac{|x_1| + |x_2| + \cdots + |x_n|}{\theta} = |1+Z_1| + \cdots + |1+Z_n|.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{|X_1| + |X_2| + \cdots + |X_n|}{b_{12,\,1-\frac{\alpha}{2}}}, \frac{|X_1 |X_2| + \cdots + |X_n|}{a_{12,\,\frac{\alpha}{2}}} \right).$$

Pivot 13. $P_{13} = \dfrac{|x_1|^k + |x_2|^k + \cdots + |x_n|^k}{\theta^k} = |1+Z_1|^k + \cdots + |1+Z_n|^k, \quad k > 0.$

A $100(1-\alpha)\%$ C.I. for θ is

$$\left(\frac{(|X_1|^k |X_2|^k + \cdots + |X_n|^k)^{1/k}}{\left(b_{13,\,1-\frac{\alpha}{2}}\right)^{1/k}}, \frac{(|X_1|^k + |X_2|^k + \cdots + |X_n|^k)^{1/k}}{\left(a_{13,\,\frac{\alpha}{2}}\right)^{1/k}} \right).$$

In fact, it is very difficult to find the exact distributions of some of these pivots. For example, the distribution of the pivots P_{10} has very complicated form (see Springer, 1979). In spite of we can obtain the exact distributions to the pivots P_4 and P_5, we have another problem in finding closed forms to their quantiles as well as the confidence interval of minimum expected length. Therefore, we restrict our study to a comparison of these pivots via simulation. Hence a computer simulation is needed to obtain these quantile. For this, we prefer to rely on the computer simulation to find these quantiles instead of reporting the exact distributions. To obtain the distribution quantiles of P_4 we present the following algorithm:

1. Input: Parameter θ, Sample Size n
2. Simulate a random sample of size n say $Z_1, \cdots, Z_n$ from $N(0,1)$
3. Find $L = |1 + Z_{(n)}|$, where $Z_{(n)} = \max(Z_1, \cdots, Z_n)$
4. Repeat the Step 2-3 5000 times to get $L_1, \cdots, L_{5000}$
5. The pivotal quantiles are appromximated by the sample quantiles of $L_1, \cdots, L_{5000}$.

Similar algorithims can be designed to obtain the distribution quantiles distributions of the pivots $P_5, \cdots, P_{13}$.

3. OTHER RELATED PIVOTS

The following pivots could be also used to construct confidence intervals for the normal distribution

1. $\max_i \Phi\left(\frac{X_i - \theta}{\theta}\right) \sim \text{Beta}(n,\ 1).$
2. $\sum_{j=1}^{n} -2\log\left(\Phi\left(\frac{X_i - \theta}{\theta}\right)\right) \sim \chi^2_{2n}.$
3. $\sum_{j=1}^{n} \log\left(\frac{\Phi\left(\frac{X_j - \theta}{\theta}\right)}{1 - \Phi\left(\frac{X_j - \theta}{\theta}\right)}\right) \sim W$

where W is a random variable which results from convolving n independent standard logistic distributions. In fact, these pivots are not welcomed, since they produce intervals which contain negative values, i.e., outside the range of θ.

4. NUMERICAL COMPARISON

To compare the 13 confidence intervals that we obtained in the previous section, we conducted a simulation study to obtain the mean and the standard deviation for the length of each interval. Without loss of generality, we assumed $\theta = 1$. For if $X : N(\theta, \theta^2)$ with $\theta = \theta_0 \neq 1$ then $\frac{x}{\theta_0} : N(1, 1)$. For an interval of length L we simulated a sample of size $n = 5000$ observations, say $L_1, \cdots, L_{5000}$, from the distribution of L. Then, these observations are used to obtain the expected length and the standard deviation of L as follows:

$$E(L) \approx \bar{L} = \frac{1}{5000}\sum_{j=1}^{5000} L_j$$

and

$$\sigma_L = \sqrt{\frac{1}{4999}\sum_{j=1}^{5000}(L_j - \bar{L})^2}.$$

These two quantities are used to compare the 13 confidence intervals (Table 1).

5. DISCUSSION

Coverage probability, mean and standard deviation of the interval length are two criteria to compare confidence intervals (Kang and Schmeiser, 1990; Jordan and Krishnamoorthy, 1996). In our simulation, we fixed the coverage probability of all intervals to be 95%. So, and according to other criterion, the smaller the expected

TABLE 1. - *Mean and standard deviation of the interval length for $\theta = 1$, $\alpha = 0.05$*

Confidence intervals \ n	6	10	15	20	25	30
C.I.$_1$	4.4483 1.3476	2.0128 0.7978	1.3606 0.5977	1.0849 0.4975	0.9263 0.4304	0.8208 0.3871
C.I.$_2$	1.9974 1.2461	1.3916 1.0191	1.0878 0.8926	0.9238 0.8188	0.8170 0.7679	0.7404 0.7298
C.I.$_3$	2.3939 1.3643	0.9272 0.8319	0.5123 0.6125	0.3521 0.5055	0.2678 0.4397	0.2159 0.3941
C.I.$_4$	1.4265 0.4035	1.0231 0.2353	0.8427 0.1701	0.7517 0.1385	0.6907 0.1193	0.6499 0.1048
C.I.$_5$	27.2268 21.5587	24.4906 18.5436	19.186 12.8245	12.1404 7.0802	7.4863 3.9086	5.1479 2.4276
C.I.$_6$	6.8843 3.0745	3.2932 1.2255	2.099 0.6691	1.4753 0.4027	1.2621 0.3132	1.0999 0.2458
C.I.$_7$	1.7745 0.5947	1.1796 0.3023	0.9688 0.2144	0.8222 0.1598	0.7364 0.1325	0.6829 0.1147
C.I.$_8$	7.7004 3.5812	5.0116 2.1023	3.6293 1.4036	3.1289 1.1658	2.9295 1.0579	2.7677 0.9727
C.I.$_9$	5.6523 1.7579	2.5922 0.8064	1.5696 0.5411	1.0612 0.4198	0.8297 0.367	0.6035 0.3135
C.I.$_{10}$	1.8452 0.7351	1.5184 0.6066	1.4819 0.5601	1.4569 0.5287	1.5251 0.5143	1.5553 0.4994
C.I.$_{11}$	2.6098 1.2133	1.0787 0.4763	0.7953 0.3104	0.6897 0.2312	0.6369 0.1909	0.5897 0.157
C.I.$_{12}$	2.3289 0.9150	1.6356 0.5139	1.2263 0.3189	1.0153 0.2325	0.9008 0.1861	0.7984 0.1513
C.I.$_{13}$ $k = 0.5$	1.5899 0.4992	1.1188 0.2767	0.8635 0.1716	0.7375 0.1281	0.6495 0.1007	0.5858 0.0848
C.I$_{13}$ $k = 1.5$	1.3986 0.3931	0.9774 0.2115	0.7597 0.1321	0.6325 0.0966	0.5623 0.0774	0.5047 0.0631
C.I.$_{13}$ $k = 2$	1.2952 0.3339	0.8623 0.1691	0.6903 0.1098	0.5838 0.0812	0.5072 0.0622	0.4519 0.0501
C.I.$_{13}$ $k = 3$	0.9621 0.2457	0.3403 0.0663	0.1688 0.0244	0.1019 0.0137	0.0762 0.0092	0.0606 0.0068

length (length standard deviation), the better the confidence interval. The results in Table 1 show that C.I.$_3$ has the smallest expected length among the first three confidence intervals. Also C.I.$_1$ and C.I.$_3$ show in general about the same standard deviation. It can be seen that the first two intervals can lead to negative estimates of θ, while the third interval always leads positive estimates. Also, the results in Table 1 indicate that C.I.$_{13}$ shows, in general a better performance with smaller expected length and in standard deviation. This positive performance for C.I.$_{13}$ becomes clearer with the increasing of k value. On the other hand, we notice that C.I.$_5$, C.I.$_8$, C.I.$_6$, C.I.$_9$, and then C.I.$_1$ are showing worst performance in this regard. With increasing the sample size, C.I.$_3$ began to show a very exceptional good performance regarding the expected length beside somehow large (but decreasing) standard deviation. Finally, despite of study limitations to the normal distribution, it can be easily extended to the location family with a known coefficient of variation.

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RIASSUNTO

Nel presente lavoro si considera il problema della costruzione di intervalli di confidenza per una popolazione normale con coefficiente di variazione noto. Si ottengono 13 intervalli di confidenza utilizzando il metodo pivotale. Si effettua quindi uno studi simulativo per confrontare sia il valore atteso che la deviazione standard della lunghezza degli intervalli.

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