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A MULTI-CRITERIA FUZZY APPROACH FOR ANALYZING POVERTY STRUCTURE

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SUMMARY

Poverty is a fuzzy and complex phenomenon which is intrinsically multidimensional. First attempts of tackling poverty with multidimensional measures trace back to the seventies with the conceptual writings on income poverty by Amartya Sen (1976). Since then much research has been devoted to answer questions of the type: (i) Who is poor? (ii) How poor is a poor? The measure of poverty and social exclusion is certainly a key point in poverty description. While much effort has been put in the last decades to the measurement of poverty, less attention has been paid to find relations among different poverty aspects. In this paper, we start from a classical definition of the population of the poor and we employ Fuzzy Multi-Criteria Analysis to provide an attempt to relate poverty aspects to one another, which we call a 'structural representation of poverty'. Our focus is on the pattern of implications existing among different descriptors characterizing poverty aspects. We show how fuzzy relation theory and partially ordered set techniques are effective in representing complex relational structures and provide new insights into multidimensional poverty. As simple test cases the method is applied to data concerning two Italian regions based on EU-SILC database 2004.

Keywords: *Multidimensional Poverty, Multi-criteria Analysis, Poverty Structure, Ordinal Variables, Posets, Fuzzy Quasi-order Relations.*

1. INTRODUCTION

In the last decades, monitoring and reduction of poverty and social exclusion are emerging as primary goals for many national governments and international organizations. At European level, after the definition of the Lisbon Agenda (March 2000) and the subsequent European Council in Nice (December 2000), the European Commission has focused on these goals and a set of indicators, the Laeken indicators[1]

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[1] Laeken indicators are a set of common European statistical indicators on poverty and social exclusion, established at the European Council of December 2001 in the Brussels suburb of Laeken, Belgium.

(Atkinson, Marlier and Nola, 2004), have been explicitly devoted for monitoring social exclusion, which is seen as a wider concept than poverty (Sen, 1998).

The introduction of social exclusion and poverty as multidimensional concepts dates back to the seventies with the seminal work by Sen (1976). Most of the research has been devoted to define multivariate poverty measures and extend classical poverty measures in a fuzzy direction. Interesting and recent examples of multidimensional measures may be found in Alkire and Foster (2011), Chakravarty and D'Ambrosio (2006); Chakravarty, Mukherjee and Ranade (1998). Their common starting point are concepts and tools developed for mono-dimensional poverty measurement but they take into account the different levels of deprivations/exclusions along various poverty dimensions. From the fuzzy prospect, some indices like the ones defined in Cerioli and Zani (1990), Cheli and Lemmi (1995) and Lemmi and Betti (2006) attempt to overcome the rigid distinction of people into 'poor' and 'not poor', recognizing the non-crisp nature of poverty.

It is our opinion that complexity of the poverty phenomenon requires some kind of structural description, as the natural complementing part of multi-dimensional measures. With this aim, in this paper we investigate the relational pattern among different poverty facets, describing the network of logical implications among attributes characterizing poverty aspects. In our approach multidimensional poverty is thus characterized by means of a 'structure' which complements the set of measures. In order to analyze poverty in terms of its multi-dimensionality we aim at establishing a set of implications among attributes of poverty. We consider this relational set of implications a 'structure'. A 'structure of poverty' cannot be seen independently of the scenarios from where it is developed. Hence in our case we refer to two Italian geographical areas. The methodological basis of our analysis is the theory of fuzzy relations and partially ordered sets, namely Fuzzy Multi-Criteria Analysis (FMCA), which provides tools to describe complex relational structures which may be also appropriate for qualitative ordinal variables. As our scenarios we select the population of poor in two Italian regions, Lombardy and Sicily, extracted from the European panel about Income and Living Conditions (Eurostat, 2004). The application is intended as a simple test case of the method proposed.

The paper is organized as follows: Section 2 introduces and discusses the problem of poverty measurement at a conceptual level; Section 3 provides some basic mathematical definitions, needed in the following analysis; Section 4 describes the EU-SILC database and the data used for poverty analysis; Section 5 discusses FMCA; Section 6 shows its application to poverty attributes. Section 7 provides an interpretation of results and some concluding comments are eventually provided in Section 8.

2. ANALYZING POVERTY

It is hard to provide a definition of poverty that can be directly turned into a set of objective statistical indicators. As a matter of fact, assessing poverty implies some economical, statistical and even ethical choices that can be argued in many respects.

First of all, poverty can be measured in an absolute or in a relative sense. Absolute poverty refers to the lack of a minimal set of goods necessary to survive, such as food or clothing, while relative poverty measures the deprivation with respect to a standard level of well-being, accepted as appropriate in a given social context. Needless to say, in developed countries the main focus is on relative poverty.

Secondly, assessing poverty requires identifying an 'evaluation space', i.e. a set of economical and social aspects retained as relevant to represent poverty. Historically, the evaluation space has been often reduced to a single monetary dimension, usually household income. This choice is quite natural and appealing, being simple and clear; nevertheless it rules out many well-being dimensions that should be considered to give a more realistic picture of poverty and social exclusion.

Once the evaluation space is identified, the problem arises of how to classify people as poor or not poor. In the relative monetary approach, this classification is usually obtained establishing a poverty line and comparing the equivalised[2] income with such a threshold. Households whose equivalised income falls below the poverty line are classified as poor. The procedures used to set the poverty line are quite arbitrary and there is no general agreement on them. For example, Eurostat sets it at 60% of the median equivalised income (Guio, 2005) while the Italian poverty line is computed by Banca d'Italia at 50% of the median equivalised income (Eurosistema, 2008). Perhaps, the most relevant limitation of the monetary approach is that only a single aspect of poverty is taken into account, while it is widely accepted that poverty is a multidimensional concept (Lemmi and Betti, 2006). For this reason, poverty surveys tend to supplement income data with additional qualitative information about living conditions. Such data allow for a more faithful representation of poverty, however they pose some statistical problems when trying to single out consistent poverty indicators. In fact, the problem of classifying people in terms of poverty is even harder when more attributes, often qualitative, are considered in the evaluation space, due to the lack of a clear scaling and aggregating criterion. Moreover, in a multidimensional setting the crude distinction of people in poor and not poor makes little sense, since poverty appears in all its graduations and nuances, emerging as a vague phenomenon. A lot of theoretical and applicative research has been devoted to extend univariate poverty measures in a multivariate perspective and different approaches exist to this goal (Lugo, 2005; Bibi, 2005). Typically, each statistical unit is assigned a final score based on its partial scores on each considered welfare attribute; if the final score falls below a fixed reference level, the individual is classified as poor. There are many concepts to combine indicators of poverty into a single one. Some are empirical in nature, other rely on more robust approaches, for example considering the axioms proposed by Tsui (2002). Irrespective of how the aggregation of single poverty scores into a single poverty indicator is done, so-

[2] Before comparing it to the poverty line, the household income is transformed into an 'equivalised income', by means of an equivalence scale, to take into account the dimension and the structure of the household (Coulter, Cowell and Jenkins, 1994).

me kind of weighted average is usually employed, the various proposals differentiating about the adopted weighting scheme.

Realizing that poverty is a vague and complex phenomenon, great interest has arisen towards the use of fuzzy set theory (Lemmi and Betti, 2006). The basic idea is to combine different indicators (quantitative or qualitative) into a membership function that measures the degree an individual belongs to the set of poor, developing subsequent analysis using fuzzy reasoning. The critical issue in this approach is, obviously, the way the membership function is computed, especially when qualitative poverty indicators are available (Cerioli and Zani, 1990; Fattore, 2008; Lemmi and Betti, 2006). Furthermore even if these approaches represent a step towards more realistic poverty measurement, they still tend to reduce complexity. They produce one-dimensional indicators which, on one hand, may be easily communicated and understood, but on the other hand are not suitable for describing the structure of poverty.

In summary, assessing poverty is a very difficult process even from a conceptual point of view, with many ambiguities and open issues at any stage. A more appropriate representation of poverty requires dealing with its complexity, without treating it with oversimplified conceptual schemes. For this reason, in this paper, we take a different perspective. Instead of describing multidimensional poverty in terms of some point indicators, we characterize it in terms of the relational pattern across its attributes, that is our concept of 'structural representation' of the poverty phenomenon. To this aim we employ some tools of fuzzy theory to explore and represent the network of implications among different poverty facets.

3. PRELIMINARIES

In this section we collect some basic definitions pertaining to fuzzy sets (Zimmermann, 1991) and partially ordered sets (Neggers and Kim, 1988; Trotter, 1992), needed in the paper. More details can be found in the references given in the text. The reader who is already familiar with these tools may easily skip this part.

3.1 *Fuzzy theory basic tools*

Definition 1 (Fuzzy set)
If X is a collection of objects, a fuzzy set $\tilde{A}$ in X is a collection of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is called the degree of membership of x in $\tilde{A}$ and assumes values in [0,1]. In the special case of $\mu_{\tilde{A}}(x)$ taking only values 0 or 1, the set $\tilde{A}$ would be the classical, crisp set A.

Often, one is interested in those elements of X that belongs to the fuzzy set $\tilde{A}$ with a degree at least equal to a chosen value α. This introduces the notion of α-cut of a fuzzy set.

Definition 2 (α-cut set)
Let X be a set, $\tilde{A}$ a fuzzy set in X and choose $0 \leq \alpha \leq 1$. The α-cut set A_α is the set of elements of X such that $\mu_{\tilde{A}}(x) \geq \alpha$.

Note that A_α is a crisp set. Most of the following analysis about poverty relies on the concepts of fuzzy binary relations and α-cut relations.

Definition 3 (Fuzzy relation)
Let X be a set. A fuzzy relation $\tilde{R}$ is a fuzzy subset of $X \times X$. If $x, y \in X$, then $\mu_{\tilde{R}}(x, y)$ measures the degree of membership of the pair (x, y) to the binary relation $\tilde{R}$ (also the strength of the relation between x and y).

If X is a finite set with cardinality n and if its elements are labeled $x_1, \ldots, x_n$, then a fuzzy relation $\tilde{R}$ on X can be represented as a square matrix $M_{n\times n}$, where

$$M_{ij} = \mu_{\tilde{R}}(x_i, x_j), \quad x_i, x_j \in X. \tag{1}$$

As for ordinary fuzzy sets, the notion of α-cut can be given for fuzzy relations:

Definition 4 (α-cut relation)
Let $0 \leq \alpha \leq 1$ and $\tilde{R}$ be a fuzzy relation on a set X. An α-cut set R_α, of $\tilde{R}$ is the crisp relation defined by

$$xR_\alpha y \Leftrightarrow \mu_{\tilde{R}}(x, y) \geq \alpha, \quad x, y \in X. \tag{2}$$

3.2 *Partial order theory basic tools*

Definition 5 (Quasi-order, Partial Order and Poset)
A quasi-order is a binary relation that is reflexive and transitive; if it is also anti-symmetric the quasi-order is called partial-order. A partially ordered set (poset) is an ordered pair $(P, \leq)$, where P is a set and $\leq$ is a partial order relation on P.

Usually, the order relation is clear from the context, so that the poset will be simply denoted by P. Two elements a, b of P are called comparable if $a \leq b$ or $b \leq a$; otherwise they are called incomparable and we write $a \parallel b$. A linear extension of a poset $(P, \leq)$ is a complete order $(P, \leq')$ on P, that is all its elements are comparable, which preserves all the comparability relations in P. In other words, a linear extension is a complete order $(P, \leq')$ where $x \leq y$ implies $x \leq' y$. A chain in a poset P is a subset of P in which every two elements are comparable; dually, an anti-chain is a subset of P in which every two elements are incomparable. A maximal chain (anti-chain) is a chain (anti-chain) that is not properly contained in any other chain (anti-chain). We say that b covers a, denoted as $a \prec b$, if $a \neq b$, $a \leq b$ and there exists no $c \in P$ such that $a \leq c \leq b$. The binary relation $\prec$ is called the cover relation. Minimal and maximal elements of a poset are defined as, respectively, elements which do not cover and are not covered by any other element of the partially ordered set.

In this paper, we deal exclusively with finite posets (i.e. posets over a finite set). In the case of finite posets, the cover relation admits a very simple graphical representation.

Definition 6 (Hasse diagram)
A Hasse diagram (HD) is a graph representing the cover relation of a poset P. Each element of P is plotted as a different node in HD; if $x \prec y$ in P, then node corresponding to x is put below that corresponding to y and an edge is drawn between them.

If the Hasse diagram of a poset can be drawn without intersecting edges, then the poset is called planar. Thanks to transitivity of the order relation, finite posets are completely defined by the cover relation itself, so that they are represented by Hasse diagrams too. Moreover finite posets may be characterized by means of some structural parameters, particularly, width, length and dimension.

Definition 7 (Width and Length)
Let $(P, \leq)$ be a finite poset. The width of P is the cardinality of a maximal antichain; the length of P is the cardinality of a maximal chain minus one.

Definition 8
The dimension of a poset is defined as the least positive integer t for which there exists a family $\{LE_1, LE_2, \ldots, LE_t\}$ of linear extensions of P such that $P = \bigcap_{i=1}^{t} LE_i$. Such a minimal system of linear extensions is called 'generating system'.

Clearly, a poset has dimension one if and only if its Hasse diagram is linear, i.e. if it is a chain.

4. EU-SILC DATA-BASE

The European Statistics on Income and Living Condition (EU-SILC) panel is the most important source of data about income, welfare and material deprivation in EU countries. The survey is held on a yearly basis since 2004, collecting information both at individual and household level on many different socio-economic attributes, such as personal and household income, employment conditions, health conditions, possession of durable goods, arrears and difficulties on various types of payment. It is important to note that while the survey is designed to give a representation of poverty at country level, in Italy it is planned in such a way that its results are representative at regional level as well. This fact allows for the use of EU-SILC Italian data to assess similarities and differences in multidimensional poverty at sub-national level. The following paragraph describes the EU-SILC variables selected for the subsequent analysis.

4.1 *Selected variables*

Among variables (attributes) included in EU-SILC data-set, we selected some which shape different aspects of poverty (Table 1). They are all of categorical type and describe: housing appropriateness (HH040, HH050 and HH090), arrears on different

TABLE 1. - *Selected criteria*

Name (assigned name)	**Original categories**	**Recoded categories**	**Category Description**
HH040 (House)	1 = Yes 2 = No	0 = No 1 = Yes	Leaking roof, damp walls, floors, foundation, or rot in window frames or floor
HH050 (Warm)	1 = Yes 2 = No	0 = Yes 1 = No	Ability to keep home adequately warm
HH090 (Toilet)	1 = Yes 2 = No	0 = Yes 1 = No	Indoor flushing toilet for sole use of household
HS010 (Pay)	1 = Yes 2 = No	0 = No 1 = Yes	Arrears on mortgage or rent payments
HS020 (Bills)	1 = Yes 2 = No	0 = No 1 = Yes	Arrears on utility bills
HS030 (Loan)	1 = Yes 2 = No	0 = No 1 = Yes	Arrears on hire purchase instalments or other loan payments
HS040 (Holiday)	1 = Yes 2 = No	0 = Yes 1 = No	Capacity to afford paying for one week annual holiday away from home
HS050 (Food)	1 = Yes 2 = No	0 = Yes 1 = No	Capacity to afford a meal with meat, chicken, fish (or vegetarian equivalent) every second day
HS060 (Extra)	1 = Yes 2 = No	0 = Yes 1 = No	Capacity to face unexpected financial expenses
HS070 (Phone)	1 = Yes 2 = No cannot afford 3 = No other	0 = Yes or no other 1 = No cannot afford	Do you have a telephone (including mobile phone)?
HS080 (TV)	1 = Yes 2 = No cannot afford 3 = No other	0 = Yes or No other 1 = No cannot afford	Do you have a color TV?
HS100 (Wash)	1 = Yes 2 = No cannot afford 3 = No other	0 = Yes or No other 1 = No cannot afford	Do you have a washing machine?
HS110 (Car)	1 = Yes 2 = No cannot afford 3 = No other	0 = Yes or No other 1 = No cannot afford	Do you have a car?

types of payments (HS010, HS020 and HS030), capacity to afford expenses not strictly necessary (HS040, HS050 and HS060), and the deprivation of basic appliances and durable goods (HS070, HS080, HS100 and HS110). Variable categories have all been modified in order to show the same polarity towards the poverty level: the higher the score the higher the poverty. This recoding procedure leads to variables all of dichotomous type.

These variables constitute the different perspectives used to analyze the structure of poverty in two Italian regions located in the northern and southern part of Italy and span our evaluation space. The existence of socio-economic differences between geographical areas of our country is well established. The Italian literature on the topic is broad and some authors claim that the different social, political and economic development of the two parts of Italy is mainly due to different historical paths, see Ballarino and Schadee (2005) and references herein. According to dimension and economic relevance we chose to analyze Lombardy, as representative of the Northern part of Italy, and Sicily, as the most southern region (Figure 1).

To get an insight into the Italian structure of poverty a fuzzy multi-criteria approach is adopted.

FIGURE 1. - *Italy map and locations of Lombardy (grey region) and Sicily (striped region)*

5. FMCA ON POVERTY ATTRIBUTES

FMCA here employed was originally proposed by Van de Walle, de Baets, and Kerre (1995) and recently mathematically refined by de Baets and de Meyer (2003a). In our case we want to analyze poverty attributes on the basis of their incidence on poor households.

The starting point for our analysis is matrix $S_{m\times n}$, with poverty attributes on rows and households on columns. S_{ij} denotes the score of household j $(j = 1, \dots, n)$ on poverty attribute i $(i = 1, \dots, m)$. The selection of poor households is performed on an income basis and is described in Section 6.

5.1 *The fuzzy subsethood relation*

The starting point for FMCA is the measurement of the level of subsethood of one fuzzy set into another, as introduced by Kosko (Van de Walle *et al.*, 1995; Haven, 1998):

DEFINITION 9
Kosko's subsethood: Let $\tilde{A}$ and $\tilde{B}$ are two fuzzy sets in X. The Kosko's degree of subsethood of $\tilde{A}$ in $\tilde{B}$ is a binary fuzzy relation defined as:

$$\begin{aligned} SH(\tilde{A},\tilde{B}) &= \frac{\sum_{x\in X} min(\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x))}{\sum_{x\in X}\mu_{\tilde{A}(x)}} && \text{if } \tilde{A}\neq\emptyset \\ SH(\tilde{A},\tilde{B}) &= 1 && \text{if } \tilde{A}=\emptyset \end{aligned} \quad (3)$$

where $min(\cdot)$ is the minimum operator that returns the smaller of its operands.

From this definition follows that $SH(\tilde{A},\tilde{A}) = 1$ and that $A \subseteq B \Longleftrightarrow SH(A,B) = 1$ for any two crisp sets A, B.

Kosko's measure has been already employed for empirical purposes within different fields (Van de Walle *et al.*, 1995; Haven, 1998). To adapt the procedure to our specific case, a short explanation is due at this point.

Let the set $X = \{x_1, x_2, \ldots x_n\}$ be the set of poor households and $\tilde{S}_i$ be the fuzzy set in the set X defined as $\tilde{S}_i = \{(x_j, \mu_{\tilde{S}_i}(x_j))\}$, where $\mu_{\tilde{S}_i}(x_j)$ is the numerical value scored by household x_j on attribute i ($i = 1, \ldots m$). Since in our case $\mu_{\tilde{S}_i}(x_j)$ may only assume value 0 or 1, we are dealing with crisp sets S_i representing row vectors of values $(0/1)$ scored by the set of poor families on attribute i. In other words, the crisp set corresponding to attribute i is $S_i = \{(1,I_1),(2,I_2)\ldots(n,I_n)\}$ where I_j, $j = 1\ldots n$, assumes value 0 or 1 depending on the score of household j on attribute i.

In this particular crisp case it is straightforward to show, applying Definition 1, that the Kosko's subsethood measure (3) reduces to:

$$SH(S_i,S_k) = \frac{|S_i \cap S_k|}{|S_i|} \quad (4)$$

which is a kind of normalized measure of the set intersection.

In the case of poverty data Kosko's measure indicates how much an attribute is implied by another, over poor households. The higher the value of $SH(S_i,S_k)$, the higher the level of fuzzy inclusion of S_i in S_k, so that the more likely is the statement that "being poor on attribute i implies being poor on attribute k".

5.2 *Turning the subsethood relation into a fuzzy quasi-order*

To be used for comparison purposes, the fuzzy binary relation SH has to be reflexive and T-transitive with respect to a chosen T-norm (Klement, Mesiar and Pap, 2000). As aforementioned, reflexivity of SH simply comes from its definition. As far as T-transitivity is regarded, its general definition is due:

DEFINITION 10 T-transitive fuzzy relation (de Baets and de Meyer, 2003a)
A fuzzy relation $\tilde{R}$ is called T-transitive if:

$$T(\tilde{R}(x,y),\tilde{R}(y,z)) \leq \tilde{R}(x,z), \quad \forall (x,y,z) \in X^3 \quad (5)$$

where T stands for a triangular norm.

The most important example of a T-norm is the *min* operator and the T-transitivity with respect to the *min* norm is called the *min*-transitivity.

The SH relation is not *min*-transitive. However, it has been recently proven that any fuzzy relation on an arbitrary universe X has a T-transitive closure (T-closure) for any T-norm, that is the smallest T-transitive fuzzy relation including it (de Beats and de Meyer 2003a). In case of a finite universe X, as for most empirical cases and for the present paper, de Baets and de Meyer (2003a) propose an operative method, known as *matrix method*, for computing the *min*-closure. These recent results allow for transforming the fuzzy relation SH into a *min*-transitive fuzzy relation $\tilde{Q}$ which is a fuzzy quasi-order relation and to perform a consistent comparison among attributes based on it.

A comment is due at this point. The *min*-transitive closure of a fuzzy relation may not adequately approximate the original fuzzy relation. So if the *min*-transitive closure is to be used instead of the original fuzzy relation, it should be verified that the former is a good approximation of the latter. Usually, the distance between two fuzzy relations can be simply measured in terms of the L_1 norm of the difference between matrices representing them (de Baets and de Meyer, 2003b). This metric is adopted here to verify the distance between the *min*-transitive closure and the original fuzzy relation in order to verify whether the analysis looses its grasp on original data. In our particular case results are encouraging as shown in Sections 6.3 and 6.4.

5.3 *Extracting a crisp partial order from the fuzzy quasi-order*

The next step is, in a sense, a step backward from the fuzzy quasi-order relation $\tilde{Q}$ to a crisp quasi-order relation Q_α that is an α-cut of $\tilde{Q}$. Q_α is more easily interpreted and provides a rationale for comparing poverty attributes. In view of constructing a consistent comparison among attributes, the fundamental result is that any α-cut Q_α of a fuzzy quasi-order relation Q is a crisp quasi-order relation (Bandler and Kohout, 1988). Thus, for each different value of α, different order relations on equivalence classes of attributes are obtained (Haven, 1998). Each equivalence class comprises a group of attributes which shall be considered as equivalent, in the sense that they have the same crisp relational pattern with respect to all other attributes. These equivalence classes become larger as α values decrease and melt together gradually (Van de Walle *et al.*, 1995).

To understand the relevance of using α-cuts in view of poverty attributes analysis, let us observe the following. As aforementioned, the starting point of the analysis is data matrix $S_{m \times n}$ with attributes on rows and households on columns. A direct approach might have been to compare the matrix rows by means of partial order techniques. However, given the high number of households, it is unlikely that all the households poor on an attribute are poor on any other attribute. As a consequence, perfect implications among poverty attributes are unlikely and many incomparability relations would arise among them. Such relations may have no relevance nor they are always economically meaningful, since one can be interested only in the main pattern of implications among attributes. This main pattern may be considered as core information we are actually interested in, whilst the remaining part may be considered as noise. Allowing the value of α to be less than one is equivalent to state that an implication between two attributes is true on a general basis even if it is not perfectly verified in the available data. Many notable contributions for addressing the problem of noisy data in partial order theory are available (Brüggemman, Halfon, Welzl, Voigt and Steinberg, 2001; Luther, Brüggemann and Pudenz, 2000; Pudenz, Brüggemman, Luther, Kaune and Kreimes, 2000). The fuzzy relation approach allows for circumventing the problem of discarding the noise translating it into the choice of the proper α-cut value. It may be argued that this is only a way of dislocating the issue: as the value of α increases, incomparability relations among attributes are assigned more and more significance. Hence the problem is shifted to the proper choice of the α-cut which assures an adequate trade-off between discarding non informative incomparability relations and keeping all relevant order relations. In Sections 6.3 and 6.4 we employ a method which implies as little subjectiveness as possible in the choice of the critical α-cut.

Once an α-cut has been selected, the resulting crisp relation is a quasi-order ($\leq$), that is a binary relation with reflexive and transitive properties. For $\leq$ to be transformed into a partial order ($\leq'$), to allow for a consistent comparison among its elements, it suffices to introduce the following equivalence relation E:

$$xEy \Leftrightarrow x \leq y \text{ and } y \leq x, \quad x, y \in X. \tag{6}$$

For each equivalence class a representative element x is chosen and the notation for its class is $\{[x], x \in X\}$. The set of equivalence classes $\{[x], x \in X\}$ with partial order $\leq'$ is defined as

$$[x] \leq' [y] \Leftrightarrow x \leq y. \tag{7}$$

On the set of equivalence classes $[x]$ the crisp quasi-order becomes a crisp partial order.

6. ANALYZING POVERTY ATTRIBUTES IN TWO ITALIAN REGIONS

6.1 *Data preparation*

We focused our analysis on the structure of poverty among households classified as poor in terms of income, according to the criterion followed by the Bank of Italy in its income survey on Italian households. The total income of a household is first scaled into an equivalent income (Guio, 2005) to take into account for the different scale economies existing within households, which depend upon dimensions and composition of the household. The equivalent household incomes are then computed and poor households are identified as those whose equivalent income falls below 50% of the median equivalent income. We applied this procedure to the household income data from EU-SILC, extracting poor households from Lombardy (112 households) and Sicily (335 households). Among the selected poverty attributes previously described, some are discarded based on a simple frequency and missing analysis (Figures 2 and 3).

Lombardy: FREQUENCY ANALYSIS, SAMPLE SIZE = 112

			Cum.	Cum.	
		Freq.	Percent	Freq.	Percent
HH040					
	1	43	38.39	43	38.39
	2	69	61.61	112	100.00
HH050					
	1	100	89.29	100	89.29
	2	12	10.71	112	100.00
HH090					
	1	111	99.11	111	99.11
	2	1	**0.89**	112	100.00
HS010					
	1	16	21.92	16	21.92
	2	57	78.08	73	100.00
		Frequency Missing = 39			
HS020					
	1	22	19.64	22	19.64
	2	90	80.36	112	100.00
HS030					
	1	6	30.00	6	30.00
	2	14	70.00	20	100.00
		Frequency Missing = 92			
HS040					
	1	46	41.07	46	41.07
	2	66	58.93	112	100.00
HS050					
	1	94	83.93	94	83.93
	2	18	16.07	112	100.00
HS060					
	1	51	45.54	51	45.54
	2	61	54.46	112	100.00
HS070					
	1	94	83.93	94	83.93
	2	6	5.36	100	89.29
	3	12	10.71	112	100.00
HS080					
	1	111	99.11	111	99.11
	2	1	**0.89**	112	100.00
	3	0	0.00	112	100.00
HS100					
	1	98	87.50	98	87.50
	2	4	3.57	102	91.07
	3	10	8.93	112	100.00
HS110					
	1	61	54.46	61	54.46
	2	13	11.61	74	66.07
	3	38	33.93	112	100.00

FIGURE 2. - *Frequency analysis of original data for Lombardy*

Sicily: FREQUENCY ANALYSIS, SAMPLE SIZE = 355

		Freq.	Percent	Cum. Freq.	Cum. Percent
HH040					
	1	158	44.51	158	44.51
	2	197	55.49	355	100.00
HH050					
	1	207	58.31	207	58.31
	2	148	41.69	355	100.00
HH090					
	1	352	99.15	352	99.15
	2	3	**0.85**	355	100.00
HS010					
	1	46	38.02	46	38.02
	2	75	61.98	121	100.00
		Frequency Missing = 234			
HS020					
	1	125	35.21	125	35.21
	2	230	64.79	355	100.00
HS030					
	1	37	48.05	37	48.05
	2	40	51.95	77	100.00
		Frequency Missing = 278			
HS040					
	1	52	14.65	52	14.65
	2	303	85.35	355	100.00
HS050					
	1	260	73.24	260	73.24
	2	95	26.76	355	100.00
HS060					
	1	125	35.21	125	35.21
	2	230	64.79	355	100.00
HS070					
	1	258	72.68	258	72.68
	2	53	14.93	311	87.61
	3	44	12.39	355	100.00
HS080					
	1	322	90.70	322	90.70
	2	16	4.51	338	95.21
	3	17	4.79	355	100.00
HS100					
	1	332	93.52	332	93.52
	2	14	3.94	346	97.46
	3	9	2.54	355	100.00
HS110					
	1	254	71.55	254	71.55
	2	41	11.55	295	83.10
	3	60	16.90	355	100.00

FIGURE 3. - *Frequency analysis of original data for Sicily*

Attributes HS010 (Pay) and HS030 (Loan) are affected by a high percentage of missing responses, higher than 35%, for both regions and were discarded from the analysis. Furthermore, attribute HH090 (Toilet) is almost homogeneous for both regions, less than 1% of the families answered 'No'; this variable is then discarded being non-informative. The same occurs for attribute HS080 (TV), in the case of Lombardy but it is kept in the analysis for comparison purpose with Sicily, where the attribute has marginal frequency higher than 1% (4.5%). Attribute categories are recoded as shown in Table 1. Furthermore, the poor households not showing difficulties on any of the selected poverty attributes are discarded: in total 20 households for Lombardy and 7 for Sicily.

In the end of this preliminary descriptive analysis, the starting data matrices are of dimensions 10×92 for Lombardy and 10×328 for Sicily.

6.2 *The software package PyHasse*

The Fuzzy partial order analysis has been performed with a new software package named PyHasse, recently implemented and still under development using Python® language by one of the authors. Particularly, the fuzzy module of PyHasse implements the matrix method described in de Beats and de Meyer (2003a) to compute the *min*-transitive closure of a finite fuzzy relation. Further details about PyHasse may be found in Annoni, Fattore and Brüggemann (2008).

6.3 *FMCA for Lombardy households*

Applying Kosko's subsethood measure to Lombardy data, the following matrix $\tilde{D}$ is produced, representing the degree of subsethood implication among attributes:

$$\tilde{D} = \begin{pmatrix} 1 & 0.818 & 0.227 & 0.727 & 0.091 & 0 & 0 & 0.091 & 0.455 & 0.227 \\ 0.273 & 1 & 0.197 & 0.652 & 0.076 & 0.015 & 0.061 & 0.152 & 0.485 & 0.182 \\ 0.278 & 0.722 & 1 & 0.778 & 0 & 0 & 0 & 0.111 & 0.556 & 0.278 \\ 0.262 & 0.705 & 0.23 & 1 & 0.082 & 0.016 & 0.066 & 0.197 & 0.508 & 0.164 \\ 0.333 & 0.833 & 0 & 0.833 & 1 & 0.167 & 0.167 & 0.667 & 0.667 & 0.167 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0.25 & 0.25 & 1 & 0.75 & 0.75 & 0.25 \\ 0.154 & 0.769 & 0.154 & 0.923 & 0.308 & 0.077 & 0.231 & 1 & 0.538 & 0.154 \\ 0.233 & 0.744 & 0.233 & 0.721 & 0.093 & 0.023 & 0.07 & 0.163 & 1 & 0.14 \\ 0.417 & 1 & 0.417 & 0.833 & 0.083 & 0 & 0.083 & 0.167 & 0.5 & 1 \end{pmatrix} \quad (8)$$

The *min*-transitive closure $\tilde{Q}$ of the matrix $\tilde{D}$ computed by the matrix method turns out to be:

$$\tilde{Q} = \begin{pmatrix} 1 & 0.818 & 0.233 & 0.727 & 0.197 & 0.197 & 0.197 & 0.197 & 0.508 & 0.233 \\ 0.273 & 1 & 0.233 & 0.652 & 0.197 & 0.197 & 0.197 & 0.197 & 0.508 & 0.233 \\ 0.278 & 0.722 & 1 & 0.778 & 0.197 & 0.197 & 0.197 & 0.197 & 0.556 & 0.278 \\ 0.273 & 0.705 & 0.233 & 1 & 0.197 & 0.197 & 0.197 & 0.197 & 0.508 & 0.233 \\ 0.333 & 0.833 & 0.233 & 0.833 & 1 & 0.231 & 0.231 & 0.667 & 0.667 & 0.233 \\ 0.333 & 1 & 0.25 & 1 & 1 & 1 & 1 & 1 & 1 & 0.25 \\ 0.308 & 1 & 0.25 & 1 & 0.308 & 0.25 & 1 & 0.75 & 0.75 & 0.25 \\ 0.308 & 0.769 & 0.233 & 0.923 & 0.308 & 0.231 & 0.231 & 1 & 0.538 & 0.233 \\ 0.273 & 0.744 & 0.233 & 0.721 & 0.197 & 0.197 & 0.197 & 0.197 & 1 & 0.233 \\ 0.417 & 1 & 0.417 & 0.833 & 0.197 & 0.197 & 0.197 & 0.197 & 0.508 & 1 \end{pmatrix} \quad (9)$$

To verify that matrix $\tilde{Q}$ is a good approximation of matrix $\tilde{D}$, we compute the following indicator β:

$$\beta = \frac{\|\tilde{Q} - \tilde{D}\|_1}{\|\tilde{D}\|_1} \quad (10)$$

where $\|\cdot\|_1$ stands for the L_1 matrix norm. For Lombardy data, $\beta = 0.136$ revealing a good level of approximation between the two matrices. In view of the α-cut analysis, we compute β also for the sequence of $\tilde{D}$ matrices obtained assigning the value 0 to elements less than cut-off values 0.05, 0.1, 0.15... and so on until 0.95. The maximum value of the sequence of β is 0.144 and is obtained for the cut-off value 0.2. Matrix $\tilde{Q}$ and all its α-cuts can thus be used instead of matrix $\tilde{D}$ for our comparison goal.

The complete series of α-cuts is composed of 26 different values, that is all the different values in matrix $\tilde{Q}$. With such a high number of α-cuts it is of little use to depict the different Hasse diagrams of the order relations among attributes for each corresponding α-cut value, as is done in other FMCA applications (Haven, 1998; Van de Walle *et al.*, 1995). Instead we prefer to plot a relevant parameter of the Hasse diagram in relation to the corresponding α-cut that provides a criterion for selecting the value of α. The chosen parameter is the ratio between the number of incomparabilities and the number of comparabilities I/C of the Hasse diagram which correspond to the particular α-cut. The purpose is to find out a simple indicator of Hasse diagram complexity whose trend can be associated to the different α-cuts. Figure 4_a shows the curve I/C vs α-cuts for Lombardy.

The least α-cut has value 0.197 thus meaning that until 0.197 all attributes are indistinguishable, belonging to the same equivalence class. As α increases, an implication pattern emerges and the partial order induced on the poverty attributes becomes more and more complex: the number of equivalence classes increases and, correspondingly, the number of attributes within each class decreases. The analysis of Figure 4_a shows a break-point in the curve I/C vs α-cut at the value $\alpha = 0.722$, which can be considered as the critical α. From this value on the I/C value rapidly increases and this means that from this point on small changes of α values (i.e.

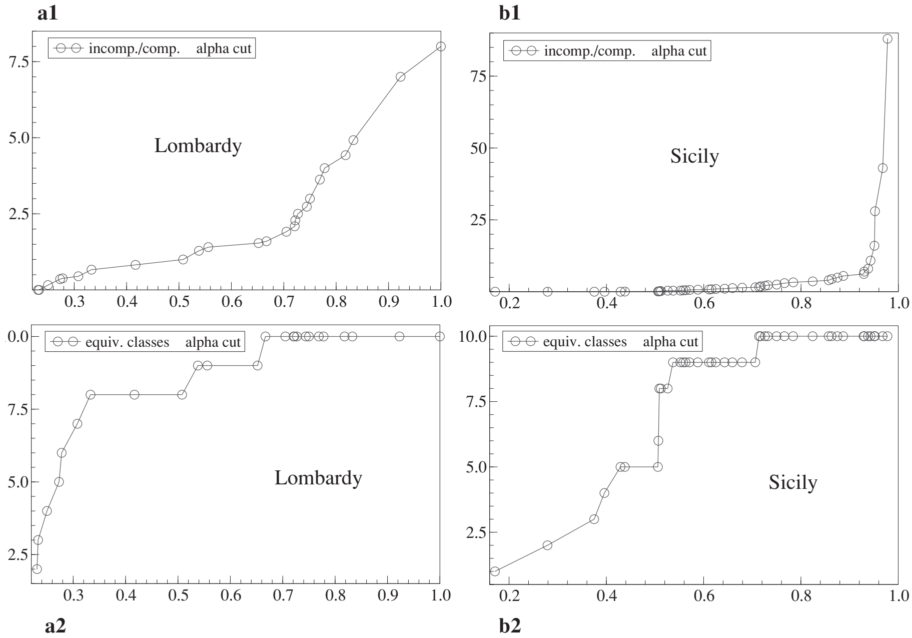

FIGURE 4. - *(a) Lombardy: I/C (a1) and number of equivalence classes (a2) as a function of α-cuts; (b) Sicily I/C (b1) and number of equivalence classes (b2) as a function of α-cuts*

small changes in the subsethood relations between attributes) induce many incomparabilities in the poset. At $\alpha = 0.722$ an implication pattern is thus revealed that is not trivial being not too sensible to noise or small changes in the subsethood relation. Taking the point $\alpha = 0.722$ as the critical point, we can plot the corresponding

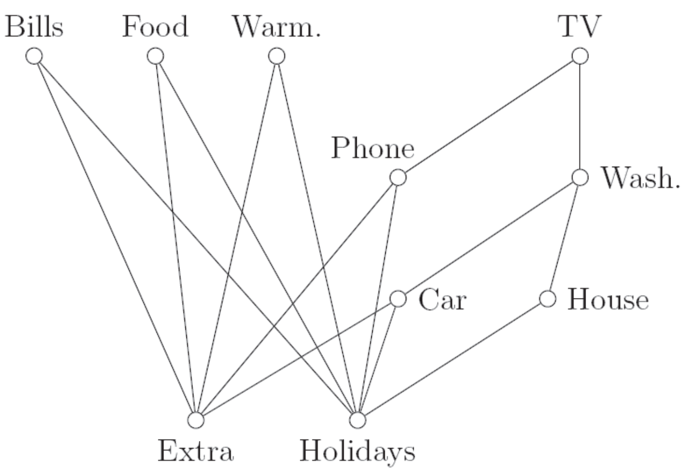

FIGURE 5. - *Lombardy: Hasse diagram for critical $\alpha = 0.722$*

Hasse diagram, revealing the implication pattern among poverty attributes. Figure 5 shows this critical Hasse diagram and graphically represents what we mean with the term structure' when analyzing the concept of poverty.

Anticipating what will be written in Section 7 we give an interpretation: for example Phone is connected downwards with two other aspects, like Extra and Holidays. This means that not affording a phone fuzzy-implies not affording holidays and not affording extra expenses as well. The diagram also shows that the aspect Phone should not be considered as substitute of Wash because both aspects imply different sets of poverty facets.

6.4 *FMCA for Sicily households*

Matrix $\tilde{D}$ representing the subsethood relation for Sicily is computed as follows:

$$\tilde{D} = \begin{pmatrix} 1 & 0.952 & 0.424 & 0.784 & 0.216 & 0.096 & 0.064 & 0.176 & 0.616 & 0.552 \\ 0.393 & 1 & 0.304 & 0.706 & 0.165 & 0.05 & 0.043 & 0.129 & 0.492 & 0.439 \\ 0.558 & 0.968 & 1 & 0.863 & 0.253 & 0.105 & 0.095 & 0.221 & 0.611 & 0.726 \\ 0.426 & 0.93 & 0.357 & 1 & 0.204 & 0.065 & 0.052 & 0.17 & 0.526 & 0.487 \\ 0.509 & 0.943 & 0.453 & 0.887 & 1 & 0.17 & 0.17 & 0.396 & 0.679 & 0.717 \\ 0.75 & 0.938 & 0.625 & 0.938 & 0.563 & 1 & 0.375 & 0.438 & 0.75 & 0.875 \\ 0.571 & 0.929 & 0.643 & 0.857 & 0.643 & 0.429 & 1 & 0.429 & 0.714 & 0.714 \\ 0.537 & 0.951 & 0.512 & 0.951 & 0.512 & 0.171 & 0.146 & 1 & 0.732 & 0.659 \\ 0.487 & 0.943 & 0.367 & 0.766 & 0.228 & 0.076 & 0.063 & 0.19 & 1 & 0.506 \\ 0.507 & 0.978 & 0.507 & 0.824 & 0.279 & 0.103 & 0.074 & 0.199 & 0.588 & 1 \end{pmatrix} \quad (11)$$

It gives rise to the following closure matrix $\tilde{Q}$:

$$\tilde{Q} = \begin{pmatrix} 1 & 0.952 & 0.507 & 0.784 & 0.279 & 0.171 & 0.171 & 0.279 & 0.616 & 0.552 \\ 0.506 & 1 & 0.506 & 0.706 & 0.279 & 0.171 & 0.171 & 0.279 & 0.526 & 0.506 \\ 0.558 & 0.968 & 1 & 0.863 & 0.279 & 0.171 & 0.171 & 0.279 & 0.611 & 0.726 \\ 0.506 & 0.93 & 0.506 & 1 & 0.279 & 0.171 & 0.171 & 0.279 & 0.526 & 0.506 \\ 0.509 & 0.943 & 0.507 & 0.887 & 1 & 0.171 & 0.171 & 0.396 & 0.679 & 0.717 \\ 0.75 & 0.938 & 0.625 & 0.938 & 0.563 & 1 & 0.375 & 0.438 & 0.75 & 0.875 \\ 0.571 & 0.929 & 0.643 & 0.857 & 0.643 & 0.429 & 1 & 0.429 & 0.714 & 0.714 \\ 0.537 & 0.951 & 0.512 & 0.951 & 0.512 & 0.171 & 0.171 & 1 & 0.732 & 0.659 \\ 0.506 & 0.943 & 0.506 & 0.766 & 0.279 & 0.171 & 0.171 & 0.279 & 1 & 0.506 \\ 0.507 & 0.978 & 0.507 & 0.824 & 0.279 & 0.171 & 0.171 & 0.279 & 0.588 & 1 \end{pmatrix} \quad (12)$$

The β coefficient equals 0.057, showing that the transitive closure approximates very well the original $\tilde{D}$ matrix. This is true also for the sequence of matrices obtained setting to 0 elements less than 0.05, 0.1, 0.15, …, 0.95. In fact, the maximum value for β is 0.128, obtained when the cut-off is 0.5.

The series of α-cuts is composed of 45 different values. As for the previous case we plot the ratio between incomparabilities and comparabilities I/C for different α-cuts (Figure 4b). From the analysis of Figure 4b two break-points seem to characterize curve: one is for $\alpha = 0.714$ and the other is for $\alpha = 0.929$. We nevertheless chose the first one since it is the lower α-cut where all attributes are separated (at this point the number of equivalence classes reaches the number of attributes) while the number of incomparabilities is not too large, thus limiting the ambiguity of the diagram. In fact, this kind of choices always implies a kind of trade-off between too much simplicity, with all attributes collapsed into one equivalence class and too much ambiguity, because of too many incomparability relations. Moreover, the first critical α-value is similar to the one of the Lombardy case and this allows for a consistent comparison between the two scenarios.

The Hasse diagram corresponding to $\alpha = 0.714$ is shown in Figure 6.

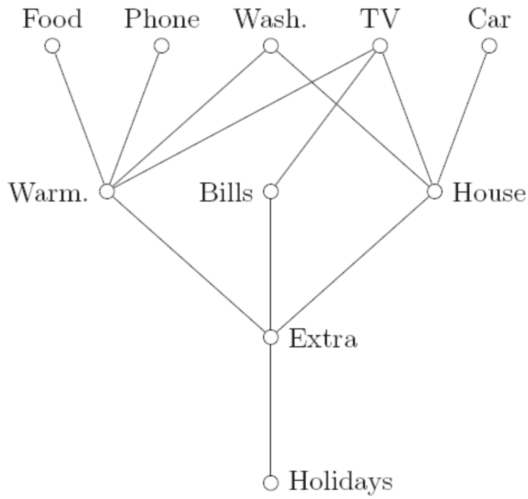

FIGURE 6. - *Sicily: Hasse diagram for critical $\alpha = 0.714$*

7. INTERPRETATION OF RESULTS

The two posets represented by the critical Hasse diagrams shown in Figures 5 and 6 can be further investigated by means of classical poset dimension theory. Recently a similar dimensional analysis of empirical posets in the social field is provided in Annoni and Brüggemann (2009).

7.1 *Reasoning of the procedure*

In Sections 5 and 6 we provide all tools to set-up the structure of poverty with respect to the two scenarios: Lombardy and Sicily. To allow for an easy interpretation of results a sketch of the steps of the procedure is outlined in the following.

1. The starting point is a measure of set inclusion between attribute profiles (sets S_i).
2. The inclusion relation is quantified by a fuzzy measure (Definition 9).

3. From the fuzzy measure a fuzzy relation is derived and, by means of α-cuts, a crisp relation is in turn derived.
4. This crisp relation forms the basis for a crisp partial order (critical Hasse Diagram).

TABLE 2. - *Generating system for Lombardy poset*

Rank	Linear ext. 1	Linear ext. 2
1	Extra	Holiday
2	Holiday	House
3	Bills	Extra
4	Food	Car
5	Warm	Wash
6	Phone	Phone
7	Car	TV
8	House	Warm
9	Wash	Food
10	TV	Bills

5. Each order relation, say attribute *a* greater then attribute *b*, is interpreted as 'not affording *a*' implies 'not affording *b*' in a fuzzy sense.
6. Eventually we arrive at a structure of poverty attributes directly derived from the critical Hasse Diagram.

In the following, separate discussions for Lombardy and Sicily are provided.

7.2 *The Lombardy case*

The Lombardy poset corresponding to the critical α-cut has dimension two, length three and width six, with maximal antichain: Bill, Food, Warm, Phone, Car, House. A system of generating linear extensions is reported in Table 2.

Being two-dimensional, Lombardy poset may be perfectly represented as a planar map, following the partial order scalogram approach as in facet theory (Borg and Shye, 1995) (Figure 7). Note that since attributes Warm, Food and Bills share the same relational pattern within the Hasse diagram, the sequence they are listed in the generating linear extensions is arbitrary. To eliminate such an arbitrariness in the planar diagram, we have mapped them into a single point.

Figure 7 may be read as follows: being deprived of an attribute having coordinates (k, ℓ) implies, in a fuzzy sense, being deprived of all other attributes represented by points with coordinates (j, h) such that $j < k$ and $h < \ell$. As a consequence, moving from the lower left corner to the upper right corner of the diagram, points representing deeper and deeper poverty condition may be found. For example, the shadowed area shown in Figure 7 includes all attributes (represented by points in the graph) implied by the deprivation of a durable good like Car, that is Extra and

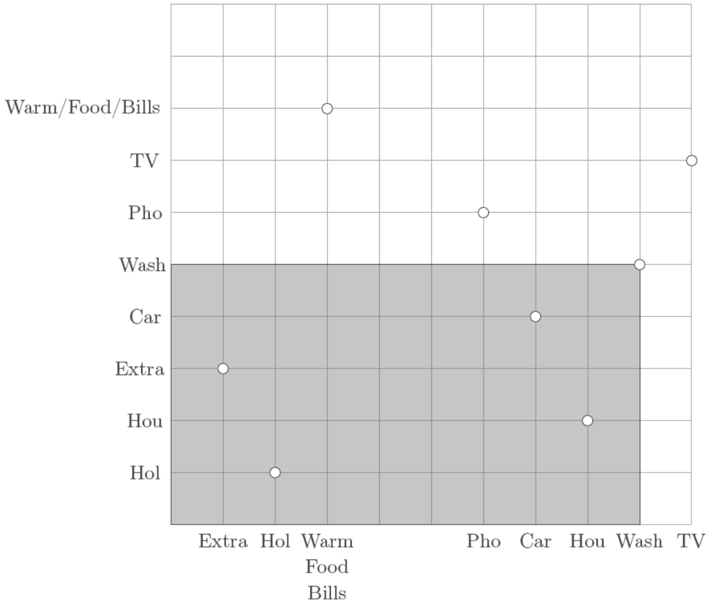

FIGURE 7. - *Planar map for Lombardy poverty attributes*

Holiday. This means that in Lombardy if a household cannot afford a car, it cannot afford also extra expenses or a one-week holiday per year. Figure 7 shows that attribute Holiday is the most implied attribute and that the lack of washing machine or TV is a serious poverty symptom. Also note that attributes seem to be quite independent from each other, as could be also guessed by the large width and the small length of the poset (Figure 5).

7.3 *The Sicily case*

A similar analysis can be pursued for Sicily. The poset corresponding to the Hasse diagram of Figure 6 has length three and width five, with maximal anti-chain including Food, Phone, Wash, TV and Car. Moreover, it has a bottom attribute, Holiday, which is implied by all other attributes. The poset dimension cannot be easily computed, but it is surely higher than one (otherwise the poset would be a linear order) and cannot exceed three, since the three linear extensions *LE*1, *LE*2 and *LE*3 reported in Table 3 are directly verified to generate the poset.

Likewise, in contrast to the Lombardy case we cannot perfectly represent the poset on a two-dimensional map. Implications between attributes are to be directly extracted from the critical Hasse Diagram.

Interpreting the three-dimensional poset shown in Figure 6, it immediately emerges that attributes Holiday and Extra are the most common features of poverty, since they are implied by every other poverty attribute. The reason for this is quite

TABLE 3. - *Generating system for Lombardy poset*

Rank	Linear ext. 1 ($LE1$)	Linear ext. 2 ($LE2$)	Linear ext. 3 ($LE3$)
1	Holiday	Holiday	Holiday
2	Extra	Extra	Extra
3	House	Bills	House
4	Car	Warm	Car
5	Warm	Food	Warm
6	Wash	Phone	Wash
7	Phone	House	Bills
8	Food	TV	TV
9	Bills	Wash	Phone
10	TV	Car	Food

clear. People rank goods and services in order of importance, starting from basic to less fundamental. Clearly, high quality food is more important than for example holidays, so that deprivation on the former is likely to imply lacking of resources for the latter. Moreover, in both maps the TV attribute may be seen as serious symptom of poverty, since it implies most other poverty attributes. Wash attribute is another poverty symptom even if to a lower extent. These two attributes can be viewed as descriptors of a deep poverty condition.

7.4 *Overall remarks*

Generally speaking, difficulties in facing unexpected expenses or in allocating money for holidays can be suffered also by people near the poverty line, who are not likely to suffer from other forms of material deprivation. Both in Lombardy and Sicily, deprivation of durable goods seems to be at the basis of other forms of poverty. This fact may be interpreted as follows. Nowadays, durable goods, like car, washing machine or television, are retained as essential assets of a household. People deprived of them are really away from the common accepted level of well-being. Moreover, deprivation on durable goods probably reveals a prolonged state of poverty. A durable good has by definition a long life, so that a household may possess it even if, at present, it has economic problems. On the contrary, lacking of durable goods is likely to reveal that hard economic conditions have been lasting for a long time.

Some differences exist between poverty patterns in the two regions. In fact, though the two critical posets for Lombardy and Sicily (Figures 5 and 6) do not differ so much with respect to macro structural features, so as width and length, they still show some relevant distinctions which may be interpreted from a socio-economical point of view. The two posets differ mainly with respect to the number and length of maximal chains and, consequently, to the presence of groups of simultaneously incomparable attributes. As far as maximal chains are concerned, it may be noted that the Lombardy poset has only three chains of length three (including four comparable attributes) while the Sicily poset has eight chains of length three. Apart from particular attributes included in the chains, this means that the latter poset is characterized by more relational implications. In other words this means that being poor in Sicily means being deficient on more aspects than in Lombardy, thus indicating a more deeply rooted poverty in the southern region than in the northern one.

8. CONCLUSIONS

Multidimensional poverty is a complex phenomenon. Its study requires developing new tools capable to overcome simple point measures, in favor of multidimensional measurement tools and structural descriptions of its features. This paper addresses the latter aspect of poverty complexity, that is its structure, from the methodological point of view. To this aim we are proposing the use fuzzy multi-criteria analysis (FMCA). The focus is on the relationships among some attributes characterizing poverty and present a procedure to represent such a relational pattern. Implication patterns produced by FMCA are posets of attributes and are represented by means of Hasse diagrams. As such, they are analyzed in terms of poset theory and different structural parameters are computed on them (dimension, width, length).

Poverty structure cannot be separated from the scenarios from which it is developed. Therefore our application is based on data from EU-SILC survey (Eurostat, 2004) relative to two Italian regions, Lombardy and Sicily. We show how the proposed approach is capable to get new insight into poverty patterns and how it may be used to characterize regional poverty in structural terms.

Interesting results come out from the analysis of the two Italian regions with structural similarities and differences between the two posets. Both for Lombardy and Sicily, deprivation of some durable goods seems to be a serious symptom of poverty, since it implies deprivation of many other attributes. The implication pattern among attributes of Sicily is richer than that of Lombardy, indicating a more structured framework in the southern region than in the northern one.

Even if the analysis leads to appealing results, further research is certainly needed both at methodological and applicative level. As an instance, at the methodological level, more research should be devoted to the choice of proper α-cuts, since the relational structure extracted by our procedure depends upon such a choice. Our approach is here to choose the critical α-cut that allows for discarding non informative differences while keeping all relevant information of the poset. Even if it seems to be quite effective, this choice is affected by some degree of subjectiveness.

From the applicative point of view, more attention should be paid to the choice of poverty attributes to guarantee a sound economic interpretation of results.

Furthermore it would be interesting to replicate such an analysis on more datasets, pertaining different countries and referring to different times. This would give the chance to assess its economic meaning and its capability to trace differences and similarities of poverty structures both spatially and temporally. In this perspective, official EU-SILC data-sets are very promising and we are going to devote some of our research to their study.

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