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ARTICOLI

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IN SENSU DIVISO *DE RE* ASCRIPTIONS AND NECESSARY EXISTENCE

1. *Introduction*

This paper is mostly concerned with an analysis of the conditions and assumptions needed to qualify the modal status of attributions of properties to one or more individuals. When, for instance, one asserts the thesis that «Any person has free will», she would presumably like to ascribe the property of 'being able to act freely' to persons. In one reading, this could be reformulated as the claim that «It is essential for a person to be able to act freely» or that «Any person has free will by necessity». Compare this claim to the thesis that «It is necessary that any person has free-will». This is a stronger reformulation of our initial thesis «Any person has free will», which would be immediately falsified by the possible existence of a completely deterministic universe. The two possible formulations of «Any person has free will» correspond to what have traditionally been called, respectively, *de re* and *de dicto* modal ascriptions. In what follows we will mostly be concerned with *de re* ascriptions and their possibility.

The distinction between modal ascriptions de re and de dicto and the analysis of their philosophical import is a recurring theme in the Western philosophical tradition. Aristotle already discussed sentences of the form

(1) It is possible for a man to sit and stand[1]

It's clear that the sentence (1), as it stands, is ambiguous: to borrow the terminology introduced by Abelard[2], by analyzing it *in sensu composito*, that is by gathering all subject-predicate components of the sentence in question under the scope of the modality, we obtain a falsehood. The *de dicto* reformulation of (1), in fact, would read

(2) It is possible for a man to sit and stand (*at the same time*)

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[1] See Aristotle, *Soph. El.*, 166a 24 - 166a 30.
[2] See Abelard, *Dialectica*, 191.1-210.19.

The sentence (2) is clearly (analytically) false. By contrast, if we divide up the two subject-predicate compounds and apply the modality only to one of them (*in sensu diviso*), we would obtain the sentence

(3) A sitting man can (*later*) stand

that is true for many notions of possibility.

The philosophical import of the distinction between *de re* and *de dicto* was also already clear to 12th century theologians and logicians: it played for instance a major role in the discussion of Augustine's account of divine knowledge by Abelard and Peter Lombard[3]. According to a classical argument dating back to Cicero, if God can know something that is not the case, then God is fallible[4]. Abelard's reply is to consider the logical form of the argument: if «God can know something that is not the case» is understood as «it is possible that ((God knows something) and (it is not the case))», then Cicero's argument is valid as it amounts to a conditional with a false antecedent. If, by contrast, the antecedent of the conditional is understood as «there is something that is known by God and it is possible that (it is not the case)», then the entire argument becomes invalid as the sentence in quotation marks becomes true when, for instance, future contingent statements are involved.

Similarly Thomas[5], in discussing the relationships between divine foreknowledge and free will, considers whether the fact that

(4) God knows that someone is standing at time t_0

could compromise the free will of human beings. According to Thomas' doctrine, in fact, God's knowledge of something makes it necessary, but surely sitting is not among someone's essential features. Again, Thomas' solution to what is seemingly a puzzle about divine knowledge is to distinguish between two readings of (4). *In sensu composito*, namely when one reads the consequence of (4) as

(5) It is necessarily true that (there is a man who's standing at t_0),

the sentence becomes true and then compatible with Thomas' doctrine – which, incidentally, is reminiscent of Aristotle's presentism. It is in fact only in a *de re* reading of (4) as

(5) God knows that there is a man who's necessarily sitting at t_0

that we reach a clearly false conclusion.

Jumping a few centuries ahead, the development of modern modal logic brought forward a powerful tool to analyze modal ascriptions and the distinction between *de re* and *de dicto* ascriptions: in quantified modal logic the distinction can be naturally understood as a scope distinction. By letting $\Box$ be a modal operator for necessity and $\forall$ a universal quantifier, the sentence $\forall x \Box B(x)$ says that the property $B(v)$ is satisfied by all objects *x by necessity*; by contrast, the sentence $\Box \forall x\, B(x)$ states that it is neces-

[3] For a complete survey of the *de re*/*de dicto* distinction in scholastic thought we refer to S. Knuuttila, *Medieval modal theories and modal logic*, in D.M. Gabbay - J. Woods (eds.), *Handbook of the history of logic. Volume 2: Medieval and Renaissance Logic*, Elsevier, Amsterdam 2008.

[4] Cf. Cicero, *De Fato and De Divinatione*.

[5] Thomas Aquinas, *Summa contra Gentiles* I, 67, 10.

sarily true that the property $B(v)$ is satisfied by all objects. Quine famously criticized the internal coherence of quantified modal logic by denying the possibility of *de re* modal ascriptions[6]: according to Quine, the truth value of the expression

(7) *necessarily*, x is greater than 7

depends on how we describe the x and not on what the variable x stands for. If x were taken to be, for instance, for «the number of planets in the solar system», the resulting sentence would be false: things were otherwise in the physical universe, the number of planets could be smaller or equal than 7. But if we consider the number of planets in the actual solar system, the resulting sentence becomes necessarily true as 9>7. What Quine is indeed denying is the very possibility of a *de re* reading of (7); in his view, we can only talk about the ways in which the number of planets in our solar systems is specified – in the case at hand a proper name or a definite description – and not about things themselves.

In this paper I assume that *de re* ascriptions are possible. More generally, I do not sympathize with the idea that logic, and modal logic in particular, is in general a metalinguistic enterprise[7] as Quine seemed to believe. As I have tried to suggest in this introduction, several champions of classical and scholastic thought also shared this view. At any rate the reader that does not share this approach may consider the content of later sections *under the condition that*, when analyzing expressions like (7), we are indeed talking directly about the object x stands for, and not a particular linguistic entity. This assumption will enable us to tackle directly some ontological implications of *de re* ascriptions by focusing on a framework for analyzing intensional notions which amounts to an expressive strengthening of quantified modal logic. The fundamental idea is to treat modal notions as predicates and not as operators. In the next section we explain how the predicative approach extends the operator approach and why it can be seen as a more natural framework to disclose the full power of modal reasoning.

2. *The first grade of modal involvement*

The approach that we are proposing is along the lines of what Quine famously called 'the first grade of modal involvement'[8]. On this approach, a modal notion is conceived as resulting is a predicate-like construction applying to a singular term: for A a sentence, N a predicate for necessity and *'A'* a canonical name for A[9], the expression $N(\text{'}A\text{'})$ is now attributing necessity to the sentence A. This way of formalizing modal ascriptions clearly diverges with the customary approach in modal logic, in which modal notions are conceived as operators that apply to sentences or formulas a yield new sentences and formulas very much like a logical operator such as negation: by keeping the terminology of the previous paragraph, the necessity operator $\Box$ applies to a *sentence A* and yields a new sentence $\Box A$. Quine thought that the way of formalizing

[6] W.V.O. Quine, *Three grades of modal involvement*, in Id., *The ways of paradox and other essays*, Random House, New York 1953.

[7] This position is reminiscent of T. Williamson, *Semantic paradoxes and abductive methodology*, in B. Armour-Garb (ed.), *Reflexion on the Liar*, Oxford University Press, Oxford 2017, pp. 325-346.

[8] Quine, *Three grades of modal involvement*, p. 157.

[9] The name-forming operation can even be take as a primitive or interpreted in our language via some coding machinery. In what follows we assume a canonical way of formalizing in the object-language the syntax of the modal language either via a direct syntax theory or via a canonical Gödel numbering.

modal ascriptions as predicates was less problematic than the operator approach as any failure of truth-functionality could be seen as ultimately boiling down to the opacity of the quotation marks or to suitable non-logical axioms[10].

However, as Quine perhaps knew already, there are clouds on the horizon. The predicate N, if governed by principles containing the following[11],

(Nec) if A, then $N(`A')$

(T) $N(`A') \rightarrow A$

would lead straight to paradox[12]. As evidence of his acquaintance with the problem, he urged that the treatment of the necessity predicate N should take the form of a metalinguistic predicate along the lines traced by Tarski in *The concept of truth in formalized languages*. There it is in fact shown how to achieve a satisfactory definition of truth satisfying (T) and the conditional form of (Nec) via a restriction of the syntactical form of sentences occurring in the principles above: if one wants to follow Tarski's approach, only sentences not containing the predicate N can appear in instances of the principles above.

But if a predicative analysis of modal ascriptions could only be given in a *typed* form, namely by resorting to a syntactic hierarchy of formulas, the proponent of the standard of modal operator logic would clearly have a striking argument against it: modal logic provides us with a powerful tool to analyze *embedded* modal ascriptions such as "It is necessary that everything could be identical to itself". But if the modal predicate could not be embedded sine contradictione, we would lose one of the very motivations for having modal notions around. This worry is clearly stated in one of the most influential handbooks in quantified modal logic available:

> But on this reading [the predicate-like approach, a.n.], there cannot be any iteration of modal operators, and as a result there is no need for a specific modal logic. There is no interpretation for the many modal logics considered earlier in the book[13].

This is clearly a hasty conclusion, for several reasons that we now briefly recall.

On the one hand, the predicate approach can be easily seen as a generalization of the operator approach: anything that can be said and done in the operator approach can be mimicked in the predicate approach. More specifically, if we focus on a first-order language L expanded with a unary operator $\Box$ – let's call this language $L_\Box$ – the usual clauses for truth in a possible world model for this language proceed inductively by starting from atomic formulas and moving to more complex formulas, including formulas of the form $\Box A$. In the predicate approach this is not in general possible: if the language L is expanded with a predicate N for necessity – let's call the resulting langauge L_N – the usual model-theoretic clauses break down at the level of atomic formulas: to assign a truth value to a formula of the form $N`A'$ we would need to know the truth value of 'A'. But 'A' is a singular term and not a formula, so it may contain

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[10] QUINE, *Three grades of modal involvement*, pp. 160 ff.

[11] These principles are the predicate version of the principles characterizing the minimal modal operator logic T.

[12] This fact is widely known as Montague's paradox.

[13] M. FITTING - R.L. MENDELSOHN, *First-order Modal Logic*, Kluwer Academic Publisher, Dordrecht 1999, p. 89.

arbitrarily complex sub-formulas: it is not necessarily a well-founded object. However, it is easy to show that by redefining the formal notion of 'being a sentence of L_N' by keeping track of the inner structure of the formula[14], one can restore the possibility of an inductive notion of truth-in-a-model for L_N. We refer the reader to the references for details, but the idea is straightforward: when coding sentences of L_N, one simply has to keep track of the occurrences of N in it and consider different levels of applications of N. In this picture, an atomic, N-free formula of the form $s=t$ would be assigned level 0, whereas a formula of the form $N('s=t')$ would be assigned level 1, and so on. The resulting picture is therefore clear: although operators can be easily defined from predicates, the converse clearly does not hold, otherwise modal operator logic would be inconsistent by Montague's paradox.

The second reason for disagreeing with Fitting's and Mendelsohn's conclusion is that a possible worlds semantics is available for L_N even when the notion of 'being a sentence of L_N' is formalized in a predicate-like manner[15]. One of the fundamental reasons of the success of modal logic is in fact the neat and fruitful semantics provided by Kripke, Hintikka, and others and that proves to be a fundamental tool to understand complex modal claims involving iterated modalities. Of course, due to paradox, there will be limitations to what one can do in the predicate-like approach: it's important, however, that the reader keeps in mind that these limitations concern the extra expressive resources provided by the predicate approach and not the ones available in the operator approach that are equally available without limitations in the predicate setting by what we have said in the previous paragraph. With this *caveat* in mind, paradox forces us either to have a restriction on the frames available in our semantics, namely on the structure of the space of possible worlds that are available in our semantics, or to the logic that one can employ in defining the model-theoretic satisfaction relation. In the latter option classical logic is given up and a (proof-theoretically) weaker logic – such as three or four-valued logic – has to be embraced. In the former case, classical logic can be kept unrestrictedly but frames containing 'loops', such as reflexive frames, cannot be employed[16].

Once the predicate approach is freed from prejudice, its full potential can be put at work. Its main advantage – as it will be also clear in the discussion below – is that metatheoretic objects and concepts can now be directly internalized in the object language. Consider claims such as 'there are a posteriori necessities', or 'there are unknowable truths'. These are fundamental theses in philosophical debates: yet, as long as we remain in effective, first-order languages such as $L_\Box$, they can only be expressed by resorting to schemata. For instance, in the case of 'there are a posteriori necessities', we could proceed by extending $L_\Box$ with an operator for a-posteriority *ap* and claim that there is a sentence A of the expanded language that such that the

[14] Such strategy was first proposed in A. Gupta, *Truth and paradox*, «Journal of Philosophical Logic», 11 (1982), 1, pp. 1-60. See also N. Belnap - A. Gupta, *The revision theory of truth*, MIT Press, Boston 1992.

[15] See V. Halbach - H. Leitgeb - P. Welch, *Possible worlds semantics for modal notions conceived as predicates*, «Journal of Philosophical Logic», 32 (2003), 2, pp 179-223 and V. Halbach - P. Welch, *Necessities and necessary truths. A prolegomenon to the use of modal logic in the analysis of intensional notions*, «Mind», 118 (2009), 469, pp. 71-100.

[16] See again Halbach - Leitgeb - Welch, *Possible worlds semantics for modal notions conceived as predicates*.

formula *apA* & □*A* holds. But it is clear that we can only relegate the existential quantifier in the metatheory. In a predicate-like treatment, no such problem arises and the claim in question is fully captured in the object language[17]. Similar consideration holds for quantification over properties and relations, which is available in operator modal logic only if higher-order resources are considered.

A third reason for disagreeing with Fitting and Mendelsohn, and the last one that will concern us, is the uniformity of the formalization of *that-clauses*. Truth is rather unanimously formalized as predicate-like, but consider sentences such as 'Not all truths are necessary'. How should one make precise this sentence, if the bearers of modal ascriptions and the bearers of truth ascriptions are of a different kind? In the predicate-like approach no such confusion arises. More generally, necessity is usually considered as a mode of truth, therefore the assumption that truth and necessity should belong to two different syntactic categories sounds at least questionable if not incorrect.

Understanding modalities as predicate-like therefore is not only not in conflict with standard modal logic, but it also offers new expressive power for a uniform and coherent treatment of modal and semantic notions. In the following section we provide the necessary tools to put this expressive framework to work and apply it to the case of *de re* necessity ascriptions.

3. *A sketch of a possible worlds semantics for* de re *necessity*

For our purposes it suffices to consider the semantics for a language with a predicate for necessity *de re* constructed following the first of the two options suggested above. We will stick with classical logic, but restrict the class of available interpretations for our language. Let's first introduce some terminology.

A *frame* is a tuple (W,R), where W is a nonempty set and R is a binary relation on W. In what follows, we can safely assume L to be sufficiently rich to formalize its syntax so to witness a satisfactory theory of bearers of modal ascriptions[18], and expand it with a *binary relation* $N(x,y)$ that we are about to interpret *de re* as «x is satisfied by the objects in the finite sequence y by necessity». The intuitive idea behind the binary predicate for necessity is that the sequence of objects $<x_1,\ldots,x_n>$ provides us with the objects – in the case at hand $x_1,\ldots,x_n$ – that are associated with the *free variables* of an open formula $A(v_1,\ldots,v_n)$: in this way we can directly quantify over the objects themselves to which we want to ascribe essential properties. For instance, the open formula corresponding to the property «… is a man» is satisfied by necessity by the sequence <Socrates> and by all other sequences whose first element is a man. The presence of sequences of objects instead of the objects themselves is needed to deal with a single predicate for necessity and not n predicates for each formula of L_N with n free variables. However, to facilitate the readability of formulas we will often write sequences by making explicit the objects in it and quantifying over them: e.g. we will write $<x>$ for the finite sequence containing x. In addition, L may contain further contingent vocabulary. Since from now on we will

[17] Similar consideration holds for quantification over properties and relations, which is available in operator modal logic only if higher-order resources are considered.

[18] For the sake of definiteness, this can be achieved by allowing L to contain the language of arithmetic $\{\underline{0},S,+,x\}$: the structure of natural numbers is in fact isomorphic to the structure of strings constructed from a finite alphabet. See J. Corcoran -W. Frank - M. Maloney, *String theory*, «The Journal of Symbolic Logic», 39 (1974), 4, pp. 625-637.

only deal with the language just introduced, let us call it again L_N. The elements of the set W will thus be *standard* models of L.

To obtain the required interpretation of $N(x,y)$, we require R to be converse-well founded on W, that is such that every subset $X=\{w_0,w_1,w_2,\ldots\}$ of W contains an element w_j such that there is no v in X such that w_jRv holds. A model for L_N is then defined as a tuple $M:=(W,R,I,D,F)$: D is a function that sends each world w in W to a nonempty set $D(w)$, the domain of a world, which we assume to always include the natural numbers or equivalently the domain of a standard model of L; I is an interpretation function for the standard and contingent vocabulary, e.g. sending the numeral $\underline{0}$ to the number 0 and, say, a predicate $Cat(x)$ to the property of being a cat; and F a function that takes a world w and returns a pair (a,b) in $Form_{L_N} \times Seq$ (where crucially $Form_{L_N}$ stands for the set of formulas of L_N and Seq is the set of finite sequences from variables of L_N to the domain of the frame, that is the union of all $D(w)$ for all w in W) such that

(8) M satisfies $N(\text{‘}A(v)\text{’},s)$ at w relative to a (in short $M\ sat_{w,a}N(\text{‘}A(v)\text{’},s)$) if and only if for all worlds u: if wRu and all elements of the sequence b assigned by w and a to s (in symbols $<x>^{a,w}$) are in $D(v)$, then also $M\ sat_{u,b}A(v)$

The clause (8) may look discomforting, but the idea behind it is very simple: we require the formula $A(v)$ to be satisfied by the objects in $D(w)$ corresponding to the sequence s by necessity if and only two connected conditions are met for all worlds u accessible from w: (i) the relevant objects in the sequence of $D(w)$-objects are also in $D(v)$; (ii) in turn these objects satisfy $A(v)$ at u. The other clauses for, atomic formulas, connectives, and quantifiers do not diverge from the ones usually employed in standard quantified modal logic with varying domains[19]. The notation $<x>^{a,w}$ deserves some more explanation: it denotes the result of interpreting the object linguistic sequence $<x>$ with the variable assignment a in such a way that the resulting sequence assigns an object in $D(w)$ to x.

An important remark on variable assignments is in order, given the importance that they will have in what follows: clause (8) forces a link between metatheoretic variable assignments and object linguistic finite sequences. It is worth then reminding the reader that, although variable assignment are *infinite objects* – that is functions from the set of variables to the domain of the frame –, they can be represented by *finite objects* by focusing on the relevant part of a variable assignment needed to evaluate a formula – e.g. its initial part.

The reader acquainted with standard operator quantified modal logic would immediately recognize the similarity of the setting just introduced to the possible worlds frames for quantified modal logic with varying domains. The crucial difference lies only in the fact that the predicate-like, expressive treatment of necessity force us to refine the tools needed to assign an extension to necessity. Given this similarity, it is clear that in general the Barcan Formula and its converse do not generally hold in our semantics[20].

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[19] See again Fitting -Mendelsohn, *First-order Modal Logic*, § 4.7.

[20] The Barcan formula is the sentence

$$\text{(BF)}\ \ P(\text{‘}\exists xA\text{’}) \rightarrow \exists xP(\text{‘}A\text{’})$$

The controversy behind it lies is the fact that it rather explicitly forces possible existence into actual existence. The controversy behind its converse can be seen by looking at its instance

$$\text{(CBF)}\ \ N(\text{‘}\forall x\exists yx = y\text{’}) \rightarrow \forall xN(\text{‘}\exists yv = y\text{’},< x >),$$

At any rate, is a remarkable fact concerning converse well-founded frames that they always admit a suitable function F satisfying (8). Other well-known frame configurations are not allowed in full generality: for instance, as we have hinted at in the previous section in considering Montague's paradox, reflexive frames satisfy unrestrictedly a predicative version of the axiom (T) and therefore do not admit an evaluation function F enjoying the property (8).

This is all we need to move forward and consider the applications of this semantics to the classical dispute on universals.

4. *Predicate-like necessity and* possibilia

We now move to the main focus of the article: we will see how some ontological assumptions that are not fully specified and often taken for granted in operator modal logic become explicit and harder to be overlooked in the case of our predicate-like approach.

4.1. *Metalinguistic existence*

Let us introduce the problem we are concerned with by means of a simple example. Consider the sentence

(9) A person might not exist

We can analyze (9) via formalization in L_N as follows:

(10) $\forall x\big(H(x) \rightarrow P(`\sim \exists y\, y = v', < x >)\big)$

where $H(x)$ stands for 'x is a person', ~ is a symbol for negation, $P(x,y)$ is a predicate for possibility defined as $\sim N \sim (x,y)$. Assume that in the world w there is at least one person; then, by (8), (10) is satisfied by a model M for L_N at w for an assignment a if and only if[21]:

for every assignment b such that $b(x)$ is in $D(w)$ and differing from a only in what they assign to x: if $b(x)$ denotes a human at w, then there is a world u accessible from w with the only element of $<x>^{b,w}$ in $D(u)$ such that no assignment c at u differing from $<x>^{b,w}$ in what it assigns to y at most is such that $c(y)$ is identical to $c(x)$[22].

Now if there are any persons at all, (10) would be false in our semantics. The reason is simple: we consider models in which the only possible world accessible from the actual does not contain the (arbitrary) person we are referring to in the actual world. The actual person represented in our example by the individual $b(x)$ in the domain $D(w)$ of the world w cannot appear in $D(u)$ and therefore the entire formula comes out as false.

This clearly contrasts with the standard approach in operator modal logic. By translating (9) in the language with modal operators, we soon realize that the corresponding sentence

(11) $\forall x\big(H(x) \rightarrow \Diamond \sim \exists y\, y = x\big)$,

which converts an innocuous logical truth into an endorsement of necessitism (see § 4 below).

[21] From now on we omit reference to a model M for L_N and focus only on worlds and assignments.

[22] For simplicity, here we assume that identity is interpreted in the domain of the model.

where $\Diamond$ is a modal operator for possibility, is true under natural assumptions. The fundamental reason of the asymmetry is that, in standard varying domain semantics, an assignment $c(x)$ picking an object in $D(w)$ but not in $D(u)$ still makes sense at an accessible world u, even if $c(x)$ is not in $D(u)$. We can simply say that there is nothing at u that is identical to $c(x)$ that, as it happens, is the same object as $b(x)$ at w.

Given the asymmetric treatment of (10) in the two approaches, a natural question comes to mind: if standard operator modal logic yields results that harmonize with our modal intuitions, isn't the falsity of (10) under natural assumptions a good argument to avoid a predicate-like treatment of modal notions? The answer is of course an emphatic *no*. First, as we have seen above, there are good reasons for endorsing a predicate-like treatment linked to the expressive resources of the object language. Second, and more importantly, there is nothing *specific* about the predicate approach that leads to the falsity of (10). It's only the specific semantics we have chosen that forces (10) to come out as false. That is, the falsity of (10) is linked only to our stipulation that '*A(v)* holds by necessity of *o*' requires *o* to exist all possible scenarios we are considering. An easy modification of our semantics along the lines of the one for standard operator modal logic would yield the truth of (10) under minimal assumptions. The first reaction to the falsity of (10), in fact, could be to follow the mainstream of quantified operator modal logic and modify (8) to

(12) $M\ sat_{w,a}\ N(`A(v)',s))$ iff for all worlds u: if wRu, then also $M\ sat_{u,b}\ A(v)$, where b stands for sequence $s^{a,w}$ assigned by w and a to s.

Therefore there is nothing special about the semantics for quantified modal logic that cannot be mimicked in the predicate-like approach. But it is important to ask *why* I introduced a semantics that leads to the falsity of (10). I intend now to argue that this semantics fits better the correspondence between metatheoretic and object-theoretic assignments forced by the predicate-like approach to *de re* necessity.

To support my thesis, let us consider the modified clause (12). Initially, it may sound even plausible: after all, we are not stipulating anything concerning *what there is* in the domain of u, but only about what can be said there. In the case of operator modal logic, assignments are usually understood as functions from the variables to the *entire domain of the frame*, that is the union of all $D(w)$ for w in W; there, when we talk about what's true at an accessible world u, we do so from a highly metatheoretic perspective and consider the pair $<x,c(x)>$ as always meaningful *in the metatheory*, although $c(x)$ does not exist at u. From the technical point of view, of course, this assumption is perfectly legitimate: by Tarski's theorem, the semantics for $L_\Box$ is given in a metalanguage (or a metatheory), which is in fact a part of mainstream mathematics, e.g. a portion of set theory, and sequences live in this mathematical universe.

But the predicate-like view forces us to consider assignments as wordly entities as they are present in the object language. In fact, it seems intuitively clear that, if there are no objects satisfying a certain property, the relation in question *is not* satisfied. But in our possible worlds semantics we are now analyzing claims like 'for a given x, it is possible that $A(x)$' as saying that 'there is a possible world u in which $A(v)$ is satisfied by the sequence <*x*>'. But how can a sequence of objects exist, at some world u, if the objects in the sequence do not exist? Consider the following example[23]:

(13) For any object it is necessary that, given another object, either it is possible that they are identical or it is impossible that they are identical.

(13) can be formalized as

(14) $\forall x N\big(\text{`}\forall y\big(P\big(\text{`}v_1 = v_2\text{'} < v_1, y >\big) \vee \sim P\big(\text{`}v_1 = v_2\text{'} < v_1, y >\big)\big)\text{'}, < x >\big)$

Now (14) immediately entails that

(15) $\forall x N\big(\text{`}\forall y \exists s\, s = < v_1, y >\text{'}, < x >\big)$

However, (15) in turn entails the claim that for any actual object and any object that can be quantified over in *all possible worlds*, the resulting sequence exists everywhere in the logical space. Since (14) is true even in the modified semantics mimicking standard operator modal logic, also (15) is true in it, and therefore sequences of *any* objects exist necessarily, including at the world u above. But this contradicts the analysis of (10) based on (12), in which we require some sequences not to exist at the world u. This is why I claim that (12) is not a satisfactory semantic condition, and why (8) has been chosen.

Nonetheless the fact that (10) comes out as false in our preferred semantics is still something to be dealt with. In the next section we will see that there are different ways to reconcile our semantics with the intuitive truth of (10).

4.2. *Forms of necessitism*

The discussion of the example (10) above brings us to an elegant, although perhaps ontologically committing, way out of the problems encountered above. In its most powerful formulation, necessitism holds that what exists does so necessarily. In our modal language, necessitism can be formulated as

(NEC) $N\big(\text{`}\forall x N\big(\exists y\, y = v\text{'}, < x >\big)\text{'}, <>\big)$

In our formulation of (NEC) the external finite sequence is not doing any work, as it applies to a sentence: this means that there are no variables to which objects have to be assigned and any sequence, including the empty sequence < > with no elements, would be equally good in evaluating the sentence. An immediate consequence of (NEC) is therefore that all domains of our frame are identical, that is for any w,u in W, $D(w)=D(u)$. Necessitism has been recently advocated by Timothy Williamson in his influential book *Modal Logic as Metaphysics*[24]. Williamson's defense relies on a form of inference to the best explanation: necessitism provides the simplest and more fruitful explanation of the structure of modal space and of our possibility of talking about it. In a sense, one can read what follows along similar lines. We now in fact introduce possible ways to reconcile (10) with forms of necessitism. But we will only consider

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[23] Examples like the one discussed had already some space in the literature, for instance in G. BEALER, *Universals*, «Journal of Philosophy», 90 (1993), 1, pp. 5-32.

[24] T. WILLIAMSON, *Modal Logic as Metaphysics*, Oxford University Press, Oxford 2013.

(NEC) as one specific form of necessitism, and consider the latter as a general thesis according to which *some sufficiently encompassing classes* of objects exist necessarily. The thesis (NEC) will thus be a radical form necessitism in our view.

We have already encountered the first form of necessitism that we want to discuss. We may call it *metatheoretic necessitism*. It is the thesis that, no matter what the domain $D(w)$ of our worlds may contain, it is always meaningful and possible to talk about and make use of functions from variables to the entire domain of a model – that is the union of all $D(w)$ for w in W, even in worlds that do not contain the projections of those functions. In this way thesis (10) and would simply come out as true. To embrace this form of necessitism, we do not need to embrace exotic views from the technical point of view. We could in fact simply modify our semantics in the way described in (12) above; that is we only need to adapt to the predicate-like approach to modal notions the standard approach followed in varying domain semantics for operator modal logic. From the philosophical point of view, however, serious problems arise. As we have seen, metatheoretic necessitism considers sequences of possible objects as existing *ante-rem* in the modal space. Moreover, to support metalinguistic necessitism, one would have to defend on the one hand the existence of pairs of objects sitting in the logical space; on the other, she would have to endorse a form of actualism. There are objects that do not actually exist although they can form existing collections. Metalinguistic necessitism therefore faces a challenge of internal incoherence: pushing in the metatheory what is patently denied at the object-language level.

A second form of necessitism may arise from the following observation. In our semantics, just as in standard operator semantics, formulas are evaluated at a *designated* world that is often called the *actual* world. A tentative answer to the problematic analysis of (10) might then be to consider the existence of the sequence <$c(x)$,…> not problematic because from the perspective of the designated world, there seems to be nothing wrong in assuming its existence. After all $c(x)$, being identical to $\langle x\rangle^{u,b}$, has to exist at w for the entire formula (10) to come out at false. Therefore, in the clause (8), we could require that objects in variable assignments need only to exist at the designated world. This strategy may give rise to a position that we label as *designated-world meta-theoretic necessitism* (DMN). Essentially, it holds that it is meaningful and always legitimate to talk about metalinguistic constructs that involve linguistic entities and objects in the domain of the designated world. In the case of thesis (10), (DMN) would entail that, although the object $c(x)$ does not exist at the world u, the pair <$x,c(x)$> is available in our semantics because there is nothing wrong in assuming, from a meta-theoretic perspective, that tuples concerning actually existing objects do exist. (DMN) seems therefore not passible of the charge of internal incoherence attributed to meta-theoretic necessitism. What is necessarily existing are essentially mathematical constructions that are rooted in the actual world. Of course a philosophical objection to (DMN) would presumably be along the lines of questioning the sense in which these constructions exist necessarily as it now seems to be completely detached from the very notion of necessity we are investigating in our semantics.

Luckily enough, there is even a more simple argument to cast serious doubts on (DMN). It relies on the analysis of intuitive truths containing iterated modalities such as

(16) It is necessary that everything could be identical to itself

The formalization of (16) in L_N is

(17) $N\left(\text{`}\forall y\left(P\left(\text{`}v_1 = v_1\text{'}, < y >\right)\right)\text{'}, <>\right)$

In (17) again we can employ the empty sequence since $\forall y$ (P('v_1=v_1', <y>)) is a sentence. By the clause (8), the sentence (17) is true at a world w relative to a variable assignment a if and only if

for all accessible worlds u and for all assignments b at u, there is a world z accessible from u such that all elements of $<v_1>^{b,u}$ are in $D(z)$ and $<v_1>^{b,u}(v_1)=<v_1>^{b,u}(v_1)$.

It should be by now clear that the truth of (17) depends on the fact that any object in worlds accessible from the actual to exist necessarily. The assignment $<v_1>^{b,u}$ at z in the example above, in fact, for (17) to be true, is such that its element $<v_1>^{b,u}(v_1)$ should be in $D(z)$. But in our semantics, even in the modified form in which we assume actual objects to appear into assignments relative to every world, we are not guaranteed that the arbitrary object $<v_1>^{b,u}(v_1)$, standing for an arbitrary object existing at any accessible world, exists at z. Therefore also (DMN) is problematic.

Finally we consider what happens if (NEC) is true. Unsurprisingly, the problems we encountered in evaluating (10) and (17), the formalizations of the sentences «a man could fail to exist» and «it is necessary that everything could be identical to itself», simply disappear. Let's consider (10). The clause (8) for the evaluation of modal ascriptions, when applied to possibility claims of the form P('$A(v)$',s), tells us that P('$A(v)$',s) is satisfied at a world w relative to a sequence a if and only if there is a world u accessible from w such that all elements of the variable assignment that is assigned from w and a to s, in symbols $s^{w,a}$, are in $D(u)$, and u satisfies $A(v)$ relative to $s^{w,a}$. But let us now reflect on what this condition tells us in the case of (10) and under the assumption that (NEC) is true. In evaluating (10), we started with the trivial assumption that there are people[25]. This means that, at a world w and relative to some assignment a, all elements of the sequence $<x>^{w,a}$, that in the case at hand reduce simply to the object assigned by a to x at w, is in $D(w)$. Even more simply, the object $<x>^{w,a}$ is a person in the actual world. But if (NEC) is true, the domains of all possible worlds coincide. Therefore $<x>^{w,a}$ will also be in $D(v)$. The reader might have noticed, however, that although the sentence (10) does not come out as false in our semantics because of the lack of suitable variable assignments, now it does so because of (NEC) itself! Isn't this unsatisfactory, given our assumption, in §4.1, of the intuitive truth of (10)?

The literature on the metaphysics of modality offers several strategies to reconcile (NEC) with the intuitive truth of (10). An elegant solution, which has received great attention in the recent literature[26], is to keep domains constant but relying on the distinction between concrete/nonconcrete existence. Consistently with the truth of (10), the quantifiers at a world w in our frame range over all that could possibly exist. However, each domain $D(w)$ contains both objects existing concretely and objects that exist nonconcretely: in the specific case of (10), the distinction between concrete and nonconcrete existence would entail that, for any person *concretely existing at the*

[25] Clearly this is also the reason why (10) could never come out as trivially true.

[26] See again WILLIAMSON, *Modal Logic as Metaphysiscs*, and B. LINSKY - E. ZALTA, *In Defense of the Contingently Nonconcrete*, «Philosophical Studies», 84 (1996), pp. 283-294.

actual world, there is a possible world in which she doesn't *concretely exist*. (10) could then be reformulated as

(9*) A person might not *concretely* exist

To suitably formalize (9*), we might need to resort to a predicate $E(x)$ ranging over objects that concretely exist at the designated world, to obtain:

$$(10^*)\quad \forall x\Big(H(x)\rightarrow P\big(`{\sim}\exists y\big(E(y)\,\&\,y=v\text{'},<x>\big)\big)\Big)$$

Unlike (10), the sentence (10*) comes out as true in the modified semantics we are now suggesting: in this framework, domains contain everything there could be, that is the domain of each world $D(w)$ coincides with the domain of the entire model, the union of all $D(u)$ for u in W. However, $E(x)$ now ranges over what's concretely existing at the designated world.

Of course the case of people contrasts sharply with the case of necessarily abstract objects such as mathematical objects, but not with the case of objects such as *mixed entities* such as collections containing both concretely and nonconcretely existing objects. Let us again consider a case of iteration of modal notions:

(18) Everything is such that it is necessary that every object could be identical with it.

The formalization of (18) in L_N is:

$$(19)\quad \forall x N\big(`\forall y P\big(`v_1=v_2\text{'},<v_1,y>\big)\text{'},<x>\big)$$

In our semantics, (19) comes out as true at w and relative to a variable assignment a if and only if

for every b differing from a only in x and for all u accessible from w in which all elements of $\langle x\rangle^{b,w}$ exist: for all assignments c differing from $\langle x\rangle^{b,w}$ only in y, there is a z accessible from u with all elements of $\langle v_1,y\rangle^{c,u}$ in $D(z)$ such that $\langle v_1,y\rangle^{c,u}(v_1)$ is identical to $\langle v_1,y\rangle^{c,u}(y)$.

The underlined parts of this semantic analysis are the crucial ones: for (19) to come out as true, we need to assume that both $\langle v_1,y\rangle^{c,u}(v_1)$ and $\langle v_1,y\rangle^{c,u}(y)$ exist at z. The assumption of the thesis (NEC) makes (19) true under natural assumptions, but this is crucially because, at any world, we can form collections of objects of any kind: $\langle v_1,y\rangle^{c,u}(v_1)$ could for instance stand for an object concretely existing at w but nonconcretely existing at z, whereas $\langle v_1,y\rangle^{c,u}(y)$ could stand for an object which is concretely existing at u but not at z. In sum, the strong form of necessitism encompassed in (NEC) dissolves the problems encountered in the analysis of the claims above by labeling as existent everything that could be, including necessarily abstract objects, contingently concrete and nonconcrete objects.

This aspect of (NEC) brings us to a final point. I started by highlighting the fact that a predicate-like approach to modal notion forces one to express – as much as it is possible – metatheoretic objects and concepts into the object-language. A paradigmatic case is represented by assignments for object-linguistic variables. Under the assumption of (NEC), it seems to make sense to speak about collections of everything there is at any point in the modal space. This is of course clear in the case of concretely existing objects and even of necessarily abstract objects such as mathematical ones:

in the actual world, the pair <this keyboard,2> exists. But does this pair exist in a world in which there are no keyboards? According to (NEC), the answer is affirmative because the keyboard I am looking at in this moment may not be concrete, but it will still exist at an accessible world either concretely or nonconcretely. Although (NEC) offers a simple and intuitive solutions to the problems above, one might still find this situation not completely satisfactory: even if one grants the analysis of quantifiers as determining one's ontology, one should be able to say that the pair <this keyboard,2> has a different status at a world in which this keyboard concretely exists and one in which it doesn't. But can we form and talk about collections including objects that are only contingently nonconcrete? The source of the trouble – if any – may be located in the absence of individuation conditions for contingently nonconcrete objects that, unlike mathematical objects, lack uniform abstraction procedures. The challenge, that we won't take up in this paper and leave to future work, seems therefore to be to accommodate this intuitions about these 'mixed' collections of objects without giving up the simplicity and fruitfulness of strong necessitism[27].

5. *Conclusion*

Operator modal logic is a powerful and elegant tool to analyze and understand modal claims. I have argued that there is an alternative to intensional logics – that indeed amounts, from the technical point of view, to a proper extension of it – and that displays several advantages such as (i) the capability of capturing in the object-language claims involving quantification over propositions, sentences, properties, relations, (ii) the possibility of treating truth, alethic and epistemic modalities, and propositional attitudes uniformly as predicate-like notions.

To test this expressively powerful framework, I have focused on the case of *de re* modal ascriptions – necessity in particular. In the setting of modalities conceived as predicate-like, one has the freedom to capture metatheoretic constructs and notions in the object-language. As case study, we introduced a flexible semantics for *de re* necessity featuring varying domains, and focused on the modal status of collections of objects containing contingently existing objects. The standard semantics for modal logic allows such collections to exist necessarily in the logical space, even if their elements don't: the predicate-like analysis of *de re* necessity brings this phenomenon to the fore as it requires that these collections have a wordly counterpart. I have then surveyed several forms of necessitism that accommodate this problem: the most promising solution is to keep the tied relationship between the quantifiers and what there is, and to focus on the distinction between concrete/nonconcrete existence. A variant of the problem above, however, perhaps still stands: it is not completely clear how to reconcile the worries about the meaningfulness of collections including contingently nonconcrete objects and the assumption of their necessary existence.

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[27] One way to accommodate similar worries might be to disentangle the unqualified notion of existence couched in first-order quantifiers and the mode of existence proper of each object. A similar view has been proposed by S. Galvan, *Teoria classica dei possibili. Possibilismo classico e suo fondamento attualistico*, «Rivista di Filosofia Neo-Scolastica», CVIII (2016), 1 pp. 3-27. According to this view, quantifiers express a notion of existence relative to the mode of existence of a producing power, whereas the mode existence proper of an object – e.g. concrete existence w.r.t. the keyboard – is suitably expressed via a nonlogical existence predicate.

Abstract

The paper investigates some ontological implications of an approach to modal notions that properly extends intensional logic by treating modal ascriptions as predicate-like. More specifically, we first introduce a semantics that can naturally deal with *de re* necessity and possibility ascriptions and that enables one to quantify directly in the object-language over properties and relations. We then show how, in this expressive framework, hidden metaphysical assumptions of standard modal logic become explicit. We focus on the problem of the modal status of collections of objects satisfying a certain relation by necessity (or possibly): although there seem to be objects that may fail to exist, collections containing them are taken to exist at any world. We discuss several forms of necessitism that accommodate this problem.

Keywords: Modal Metaphysics, Modal Logics, Predicate Approaches to Modality